4th Grade

WCSD Curriculum Guides

Elementary Mathematics

Version 4: 2022/2023

Curriculum is one component of a larger mathematics instructional program in Washoe County School District (WCSD) for Kindergarten through 5th grade students. The purpose of curriculum guides are to bridge the district's K-5 Philosophy of Mathematics Education with the Nevada Academic Content Standards (NVACS) through a connection of the Curriculum Pacing Frameworks, instructional materials (*Bridges in Mathematics* or ***enVision**math**2.0***), research based instructional practices and clarification of the standards when necessary. The following describes a course of study for the specified grade for one year. ***ALL*** students must receive quality instruction in ***ALL*** grade level standards in one instructional year.

This guide is designed to be **used with the instructional materials** during planning. *This guide is not meant to supplant any portion of the instructional materials*. Teachers will continue to read through Units/Topics during instructional planning.

Guide language:

Throughout the guide the following language is used to describe the level of understanding expected at the lesson level. This language is found in the lesson-by-lesson section in the column labeled "Big Idea Mathematical Development".

Beginning: Indicates students initial explorations with the mathematical idea(s) explored in the lesson. ***Instruction continues*** *to the next lesson*.

Developing: Students have worked with the mathematical ideas in previous grades or previously during the year. The focus of the lesson is to connect and build student understanding. Teachers provide intensified support to students who may exhibit misconceptions, partial understanding, no or limited understanding. ***Instruction continues*** *to the next lesson*.

Secure: Indicates that students have worked previously with these ideas and are expected to be at a level of secure understanding. Students with secure understanding are able to make connections and use the mathematics in a variety of situations; yet may still struggle expanding the understanding to non-routine situations. Students who are secure may still make mistakes at times; yet these students demonstrate that they have mathematical understanding with limited if any misconceptions. Students not secure in the understanding by the end of that Unit/Topic might benefit from small group intensification on these ideas. Teachers may choose to use an **F/D/E** (**F**ormative processes, **D**ifferentiation or **E**xtension) day to provide additional instructional opportunity; yet should be cautious to not spend too long exploring these ideas to ensure students have ample opportunity for instruction to ALL of the Nevada Academic Content Standards (NVACS) for mathematics.

This lesson indicates a level of secure understanding.

NVACS (Content and Practices)	Big Idea Mathematical Development	Instructional Clarifications & Considerations
Lesson 2-1: Even and Odd Numbers		
2.OA.C.3 2.OA.B.2 MP.4 MP.5 MP.7	**Access Prior Learning:** In first grade, students had the opportunity to work with the classification of even and odd numbers. **Securing the Big Idea:** In this lesson, students are *securing* understanding that numbers can be classified as even or odd by showing numbers as two equal parts.	Students continue to build fluency with addition and subtraction facts within 20 as they construct the big idea of equivalence and the understanding that even numbers can be decomposed with doubles facts. **Topic Opener:** Consider limiting the *Topic Opener* to discussion of the *Topic Essential Question* (TE p.77), *Review What You Know* (TE p. 78-80) and the *Topic 2 Vocabulary Words Activity* with the words *even* and *odd*. Introduce remaining vocabulary words as they appear in the lessons. Post the question and student strategies on your math focus wall. **Visual Learning:** Have students make cube towers to increase understanding and engagement. Although the *Visual Learning* discusses the pattern in the ones digits for even and odd numbers, focus the conversation on defining even numbers as numbers that can be broken into two equal

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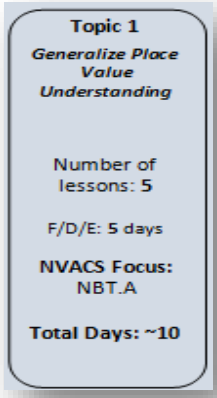
Please reference Essential Outcomes during planning.

Note:
Please e-mail Denise Trakas (dtrakas@washoeschools.net) with any questions, concerns or potential correction suggestions.

Grade 4 Topic 1: Generalize Place Value Understanding



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: K-5 Progression on Number and Operations in Base Ten (pp. 12-15)
Prior to instruction, view the Topic 1 Professional Development Video located in Pearson Realize online. Read the Teacher Edition (TE): Cluster Overview/Math Background (pp. 1A-1F), the Topic Planner (pp. 1I-1J), all 5 lessons, and the Topic Assessments (pp. 41-42A).

Mathematical Background: Read Cluster Overview (TE, pp. 1A-1F)	**Topic Essential Questions:** How are greater numbers written? How can whole numbers be compared? How are place values related? *Reference TE p. 1 and Answering the Topic Essential Questions (TE, pp. 39-40) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

1-1	1-2	1-3	1-4	1-5	Assessment

5 F/D/E days used strategically throughout the topic.

Instructional note:
This topic focuses on place value understandings and relationships. Focus instruction on Nevada Academic Content Standards (NVACS) 4.NBT.A.1, 4.NBT.A.2 and 4.NBT.A.3. Emphasis for standard 4.NBT.A is "Place-value understanding requires an integration of new and difficult-to-construct concepts of grouping by tens (the base-ten concept) with procedural knowledge of how groups are recorded in our place-value scheme, how numbers are written, and how they are spoken" for whole numbers (Van de Walle, Karp, Bay-Williams, 2010, p. 188).

As students work with number names, Van de Walle, et. al., states, "there are several variations of the base-ten language for 53- 5 tens and 3; 5 tens and 3 ones; 5 tens and 3 singles; and so on. Each may be used interchangeably with the standard name, fifty-three" (p. 189). Students will develop the understanding that one place value position to the left is **ten times greater** than the previous place value position. For example, when comparing the values of the digit 4 in the whole number 440, 400 is ten times greater than 40.

In this topic, students will also compare and round whole numbers. Rounding whole numbers is one type of **estimation** strategy. "The term estimation refers to a number that is a suitable approximation for an exact number given the particular context" (Van de Walle, et al., 2010, p. 241). Rounding is one strategy used to estimate. Number lines are useful tools to help students round numbers. Other estimation strategies include; compatible numbers, front-end methods, clustering and using tens and hundreds. Students should be able to use and recognize words and phrases for estimation like; about, approximately, close to; etc. Van de Walle, et al., (2010) states,

> Do not reward or emphasize the answer that is the closest. It is already very difficult for students to handle "approximate" answers; worrying about accuracy and pushing for the closest answers only exacerbates this problem. Instead, focus on whether the answers given are *reasonable* for the situation or problem at hand (p. 242).

Students should be able to use rounding flexibly and understand it conceptually, so it can be a useful estimation strategy (Van de Walle, et al., 2010).

Focus Math Practice 3: Construct viable arguments and critique reasoning of others
Focus opportunities for students to develop *Mathematical Practice 3* behaviors, as this is the focus of *Math Practices and Problem Solving* lesson 1-5. Reference the Teacher's Edition (pp. F23-F23A) and the Nevada Academic Content Standards for Mathematical Practice (NVACS, 2010, pp. 6-8).

This topic has five lessons with additional days for formative process (F), differentiation (D) and enrichment (E) or (F/D/E). Finally, please note that lessons 1-1 and 1-2 indicate that these are possible 2-day lessons. Additional F/D/E days were built into the 2022/2023 WCSD 4th Grade Pacing Framework so that you could take additional time to establish class routines and expectations for:

- Accessing and returning manipulatives
- Classroom discussion norms
- Mathematical Mindset (Growth vs. Fixed)
- Integrating ideas from the Math Practices and Problem Solving Handbook (TE, Vol 1, pp. F19-F36)
    - Pay particular attention to the ideas found in the *Problem Solving Guide* (p. F31)

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
millions period	*place value* *expanding form* *greater than symbol* *less than symbol* *rounding* *conjecture*

Additional terminology that students may need support with: number names, value, relationship, generalize, digit

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students developing conceptual understanding around place value understanding and relationships?"

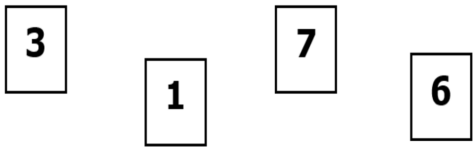
Lesson	Evidence	Look for
1-2	***Math Practice and Problem Solving*** (student work samples) Item 14	Focus CTC on the big idea: • students recognize the same digit will have a different value based on its place-value position.
1-4	***Convince Me!*** (digital platform or student work samples)	Focus CTC on the big idea: • students flexibility within place value to round. Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	***Topic Performance Assessments*** SE pp. 39-42	Use *Scoring Guide* TE pp. 39-42A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Math Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 1-1: Numbers Through One Million		
4.NBT.A.2 MP.2 **MP.3** MP.5 MP.6 MP.7	**Access Prior Learning**: In third grade, WCSD's outcome is students are to work with whole numbers in the ten thousands. **Beginning of the Big Idea:** Students will read, write and compare multi-digit numbers. Students will work with whole numbers to the millions, but may need some work around numbers less than the millions place.	**(Possible 2-day lesson)** **Day 1:** **Topic Opener**: Consider having students answer the Essential Questions before beginning the lesson. This will give you an idea of what students may know regarding place value understanding and relationships. You may also want to consider doing the *Review What You Know*, which will give you an item analysis and Intervention based on how students perform. Consider using a blank page or a separate math journal, so students have more space to explain their thinking and to include learning opportunities. For example, students may make their own place-value chart in the journal. **Solve & Share**: Consider using item 13 "MP.5 Use Appropriate Tools" or item 17 "Higher Order Thinking" for your *Solve & Share* (SE, p. 8). These problems have a higher cognitive demand and may elicit more strategies or models as you begin setting up problem solving routines. Introduce problem-solving routines, tool use and management strategies. Students should have access to, and be encouraged to use tools throughout the math instruction. **Day 2:** **Visual Learning**: In the *Visual Learning Animation,* mathematical vocabulary is introduced throughout the animation. Consider the use of a Math Focus Wall for vocabulary and emphasizing accurate mathematical terminology or language use throughout the lesson. ***-continues on next page-***

		Consider making a Place Value Anchor Chart or referring/adding to a Place Value Poster to support the *Visual Learning Animation* and *Guided Practice* portions of the lesson. This anchor chart or poster should have pictures that show place value positions and their relationships. Consider having students make and add to a place-value chart in their math journals. You will need room to add details as you progress through the lessons in Topic 1. **Guided Practice:** Consider a math discussion around item 2, to elicit knowledge students have regarding place-value understanding specifically to relationships between moving from one period to the next. **Independent Practice/Math Practices and Problem Solving:** Remember, students do NOT need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. **Note:** If item 17 was used in lieu of the *Solve & Share,* consider using item 13 as a *Quick Check* replacement. **Assess and Differentiate/Intervention Activity:** If students are having difficulty in expanded form and number names, consider using the *Intervention Activity* (TE, Vol. 1, p. 9A) with a group of students or the whole class. Additional problems can be found online under *Practice Buddy* or *Reteach page*. **Homework & Practice:** Item 11 reinforces the *Solve & Share* (item 17) and *Quick Check* (item 13). Consider using this item as part of the formative assessment process in class or for homework.
Lesson 1-2: Place Value Relationships		
4.NBT.A.1 **4.NBT.A.2** MP.2 **MP.3** MP.8	**Access Prior Learning**: In previous grades, students learned the size of the number does not change the relationship between place values within the number. **Beginning of the Big Idea:** Students will begin to explore the relationship between each place in a number. Students begin to understand the place value to the left is ten times greater than the previous position.	(**Possible 2-day lesson)** **Day 1:** **Solve & Share:** Consider using item 14 in the *Math Practices and Problem Solving* for your *Solve & Share*. This problem has a higher cognitive demand and may elicit more strategies or models as you continue setting up problem solving routines. Students should have access to, and be encouraged to use tools throughout math instruction. **Day 2:** **Visual Learning:** Consider having students make their own place value chart as you add to your anchor chart or place-value poster. Students may need some opportunity to use place value blocks to develop their understanding of larger numbers. Students were given some place-value blocks in their tool bag (concrete) but may use Teaching Tool 4 and 5 (representational) (located in *Teacher's Resource Masters Vol. 2*) to help build the big idea. **Convince Me:** Consider using the *Convince Me!* as part of the formative assessment process based on the big idea. Students need to understand the digits are the same, but the value of those digits are different due to place value position. **Another Example:** *Another Example!* may appear in the lesson. Some reiterate the mathematical focus of the *Visual Learning Animation*, while others may introduce relevant new information needed for understanding the big idea. **Assess and Differentiate/Intervention Activity:** Consider doing the *Intervention Activity* (TE, Vol. 1, p. 15A) with the whole class. Some students may write whole numbers where the same digit is next to each other (i.e., 9,972), or students may write whole numbers such as 9,989 where multiple digits are the same but in different place value positions. In doing this activity, students will continue their understanding of place value relationships and positions of the digits. ***CTC: *Math Practice and Problem Solving Item 14*** (student work samples)

Lesson 1-3: Compare Whole Numbers		
4.NBT.A.2 MP.1 MP.2 **MP.3** MP.4	**Access Prior Learning**: Students worked on comparing numbers less than 1,000 in previous grades. **Developing the Big Idea:** In this lesson, students will continue to develop place-value understanding by using strategies (example: number lines, place value charts) to compare the relative magnitude of multi-step whole numbers.	**Note**: When comparing whole numbers, be cautious as students may develop a misconception if a number is "longer" it has a greater value. This misconception may not be exposed in this topic, but as students begin their work with comparing rational numbers, fractions and decimals in later topics or grades the misconception may surface. For example, 484.03 and 484.3 students may say 484.03 > 484.3 because it is "longer", but it does not have a greater value, 484.03 < 484.3. **Solve & Share:** If students do not use a number line or place value chart to compare multi-digit whole numbers as a strategy, then consider using *Analyze Student Work* (TE p. 17, and available online under the *Solve & Share* as "Teacher Resources") to display Marco's Work (place-value chart) and Liana's Work (number line) and have students analyze and critique their strategies. **Visual Learning:** The *Visual Learning* gives students another strategy to use when comparing multi-digit whole numbers, which connects to the *Solve & Share* strategies students shared and compared. **Convince Me:** Consider using the *Convince Me!*, as it may help with students' developing conceptions (misconception). When discussing the solution with your class, make sure ***whole numbers*** is emphasized. This will not be the case when students begin work with rational numbers, fractions and decimals. **Independent Practice/Math Practices and Problem Solving:** Remember, students do NOT need to do all the problems in their student edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider utilizing the following question formats during practice: Example 1 Full Statement **Example Stem:** Joe and Kate were playing a number game with the following four cards.  The winner of the game is the person that makes the number with the greatest value. Joe made the number 6731. Using the same cards, what number could Kate make to win the game?
Lesson 1-4: Round Whole Numbers		
4.NBT.A.3 MP.2 **MP.3** MP.5	**Access Prior Learning**: In third grade (3.NBT.A.1), students estimated by rounding whole numbers to the nearest 10 or 100. **Developing the Big Idea:** In this lesson, students will estimate by rounding whole numbers to the nearest thousand, ten thousand and hundred thousand.	**Note**: Rounding is one of many estimation strategies. Ensure students understand that rounding is not separate from estimation; it is one type of estimation. **Solve & Share**: You may consider extending the *Solve & Share* by having students explain why they chose their whole numbers close to 300. When discussing the *Solve & Share*, consider using the language of underestimate and overestimate. Students can decide if the numbers they chose gave an underestimate or overestimate when rounding to 300. **Visual Learning:** Consider starting an anchor chart with estimation strategies and adding to the chart as estimation strategies arise throughout the year. **Convince Me:** Consider using the *Convince Me!* as part of the formative assessment process or as a discussion, regarding rounding and what it means to round. See the "Instructional Note" for additional information. ***-continues on next page-***

		Independent Practice/Math Practices and Problem Solving: Consider a focus on item 34 as it reiterates the discussion from the *Solve & Share*. Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** When rounding to the nearest thousand, what is the least whole number that rounds to 16,000? Enter your answer in the response box. ***CTC: *Convince Me!*** (digital platform or student work samples)
Lesson 1-5: Math Practices and Problem Solving- Construct Arguments		
4.NBT.A.1 **4.NBT.A.2** **4.NBT.A.3** **MP.3** MP.1 MP.2 MP.6	**Access Prior Learning**: In previous grades, students have constructed arguments. **Developing the Big Idea:** When constructing an argument, students should analyze the information given and use previously learned concepts to draw conclusions.	**Solve & Share:** Consider having students work on the problem independently and then have students work together to make a poster of one of their strategies or models by asking students to justify their reasoning. Next, engage students in a Gallery Walk (ELL Toolkit pg. 22) where students will analyze and evaluate other groups' responses by providing complete and clear explanations of their thinking. **Visual Learning:** The mathematical vocabulary word, conjecture is discussed in the *Visual Learning Animation*. This word should have been introduced in the 3rd grade instructional materials, however, consider further discussions regarding the idea of conjecture. **Convince Me:** Consider using the *Convince Me!* as part of the formative assessment process, or as a place to model the thinking habits for constructing arguments. Even though students have constructed arguments in previous grades, they still need to hear and see those thinking habits.

References

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Van de Wall, J., Karp, K., & Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally. Boston, MA*: Pearson

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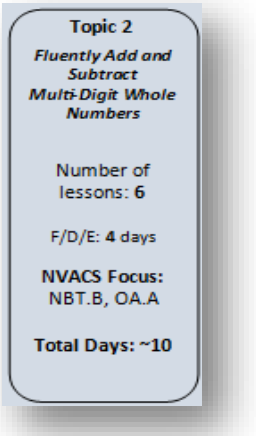
Grade 4 Topic 2: Fluently Add and Subtract Multi-Digit Whole Numbers



Big Conceptual Idea: Number and Operations in Base Ten (pp. 12-15)

Prior to instruction, view the Topic 2 Professional Development Video located in Pearson Realize online. Read the Teacher Edition (TE): Cluster Overview/Math Background (pp. 43A-43F), the Topic Planner (pp. 43I-43J), all 6 lessons, and the Topic Assessments (pp. 87-90A).

Mathematical Background:	**Topic Essential Questions:**
Read Cluster Overview (TE, pp. 43A-43F)	How can sums and differences of whole numbers be estimated? What are standard procedures for adding and subtracting whole numbers? *Reference the TE page 43 and Answering the Topic Essential Questions (TE, pp. 87-88) for key elements of answers to the Essential Questions.*

Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

The lesson map for this topic is as follows:

2-1	2-2	2-3	2-4	2-5	2-6	Assessment

4 F/D/E days used strategically throughout the topic.

Instructional note:

This topic focuses on addition and subtraction of multi-digit whole numbers. Focus instruction on Nevada Academic Content Standards (NVACS) 4.NBT.B.4 and 4.OA.A.3 (2010). Emphasis for standard 4.NBT.B uses place value understanding and properties of operations to perform multi-digit arithmetic. This topic begins a major cluster group. Topics 2-5 are clustered together focusing on 4.NBT.B. Topic 7 will come after Topic 2, which will break up this major cluster, as the focus of Topic 7 is 4.OA.B. This major cluster will continue after Topic 7 with Topic 3.

Again, the focus of Topic 2 is for students to fluently add and subtract multi-digit whole numbers by using the standard algorithm. As defined by the NVACS standards, fluency refers to "skill in carrying out procedures *flexibly*, *accurately*, *efficiently* and *appropriately*" (NVACS, 2010, p. 6). The National Council of Teachers of Mathematics (NCTM) states, "procedural fluency is more than just memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation" (2014, p. 1). When instruction focuses on memorization, students are less willing to think about numbers and their relationships and to apply and develop their number sense.

> *High achieving students use number sense and it is critical that lower achieving students, instead of working on drill and memorization, also learn to use numbers flexibly and conceptually. Memorization and timed tests stand in the way of number sense, giving students the impression that sense making is not important. (Boaler, 2015, para. 13)*

Rather, development of fluency occurs in three phases: 1) Constructing meaning and counting strategies (e.g., count on) 2) Reasoning strategies (e.g. doubles, near doubles) 3) Working toward quick recall. The third phase, quick recall is defined as allowing students to use a known fact to quickly derive an unknown fact without resorting to inefficient counting methods in about 3 seconds (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). For example, if students quickly say 5 + 6 = 11, they may have used a derived fact like 5 + 5 = 10 and 10 + 1 = 11 or 7 x 7 is 49, thus 8 x 7 is one more group of 7 so 49+7 or 56.

When assessing fluency, AVOID timed tests. Approximately one-third of students begin to experience math anxiety at the onset of timed testing (Boaler, 2014). In addition, timed tests do not tell us which strategies a student used or their level of flexibility. It is important that our instruction and assessments focus on numbers and their relationships. Kling and Bay-Williams state, "students can learn facts effectively without the use of timed testing" (2014, p. 490). Better options for assessment include student interviews, observations, journaling or quizzes based on strategies (Kling, Bay-Williams, 2014). For examples, reference "Assessing Basic Fact Fluency" and "My Fluency Progress" (Teaching Tool 30). Fluency Practice Activities are at the end of Topics 2-16 (TE, p. 83). There are online games to give students more opportunities for fluency practice in the Game Center and Center Games Lesson 2-1 and 2-5.

Along with fluency, a note of caution on developing meaning of addition and subtraction through keyword strategy instruction. Keyword strategy instruction is defined as assigning a mathematical operation to certain words. For example, terms like, each, as much, twice identify as keywords for multiplication. However, Karp, K., Bush, S., and Dougherty, B., state that, "reducing the meaning of an entire problem to a simple scan for key words has inherent challenges" (2014, p.21). Those challenges being:

- Using keywords often encourages students to strip numbers from the problem and use them to perform a computation outside the context.
- Unfortunately, many keywords are common English words that can be used in many different ways.

The focus of this topic is for students to develop the meaning of addition and subtraction. Focusing on simple keywords, rather than developing meaning through reasoning of the contexts and modeling the operations, will likely confuse students. This approach will also limit student understanding since many of the keywords for these operations overlap.

Focus Math Practice 2: Reasoning
Focus opportunities for students to develop *Mathematical Practice 2* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 2-6. Reference the Teacher's Edition (TE, pp. F22-F22A) and the NVACS (2010, SMP 2, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
variable algorithm	*commutative property of addition* *associative property of addition* *identity property of addition* *counting on* *compensation* *inverse operation*

Additional terminology that students may need support with: mental math, estimate, break apart, bar diagram, addends, sum, minuend, subtrahend, difference, regroup

*Consider using the additional terminology to label anchor charts used throughout this topic.

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "How are students applying place value understanding and the properties of operations to perform multi-digit addition and subtraction?"

Lesson	Evidence	Look for
2-3	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • students estimation, strategies and models. • use of regrouping and place value understanding.
2-5	***Quick Check*** (digital platform or student work samples) Item 5	Focus CTC on the big idea: • students solve multi-step problems with both addition (regrouping) and subtraction (across zeros). Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 87-90	Use *Scoring Guide* TE pp. 87-90A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 2-1: Mental Math: Find Sums and Differences		
4.NBT.B.4 **MP.2** MP.3 MP.6 MP.7	**Access Prior Learning**: In previous grades, students learned different properties for addition. Beginning in second and third grade, students used mental math strategies such as, breaking apart, compensation and counting on to add and subtract whole numbers mentally.	**Note:** Consider spreading this lesson over two days. If choosing to do the lesson over two days, know this is one of the F/D/E days in the WCSD suggested pacing framework. This lesson gives various estimation strategies that students may apply mentally. **Solve & Share:** The *Solve & Share* presents an *Add To Result Unknown* problem. Consider removing the "think bubble" when students are working on this problem independently. The "think bubble" gives students clues of what to do when solving this problem. Use an anchor chart to display students' strategies. Watch for students who use place value understanding, and other mental math strategies like compensation to add to the anchor chart. ***-continues on next page-***

	Developing the Big Idea: In fourth grade, students will continue using addition properties and mental math strategies to add and subtract greater whole numbers. **Look Ahead:** Estimation is a recurring concept throughout 4th grade. In this topic, consider having students estimate before finding the actual sums and differences. This helps students to determine if their answers are reasonable.	**Visual Learning:** Connect strategies used in the *Solve & Share* to the properties used in the *Visual Learning Animation*. Consider pausing after each addition property to discuss how the properties are used when adding mentally. Make an anchor chart with the addition properties so students can reference. **Convince Me:** In the *Convince Me!* students are asked to use mental math to add 150 + 2,300 + 250. Students may use addition properties (given as a sample response) to add mentally. Honor other mental compensation strategies used to solve the problem. Consider also doing the following Number String with your students (Fosnot and Uittenbogaard, 2007): 102 - 5 102 - 97 1003 – 5 1003 – 997 152 – 49 152 – 3 10,002 – 5 10,002 – 9999 Do one problem at a time and record students' strategies. Use the Number String as a formative assessment to see the strategies students use to subtract whole numbers mentally. The focus of this string is to have students use landmark or anchor numbers to support strategies; such as, break apart constant difference into two groups then add ("Think Addition") or engage in compensation strategies. **Another Example:** In the *Another Example!*, students will develop mental math strategies; break apart, counting on and compensation to add and subtract whole numbers. Consider facilitating a class discussion around the mental math strategies by having students attempt the problems on their own by posing the problems individually without the Student Edition (SE) open. Encourage students to solve in different ways. After the discussion, add the mental math or estimation strategies like breaking apart and adding-on to the anchor chart. **Independent Practice/Math Practices and Problem Solving:** Students do NOT have to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with students who may need place-value blocks or other tools to "count on" and/or to compensate.
Lesson 2-2: Mental Math: Estimate Sums and Differences		
4.NBT.B.4 **4.OA.A.3** MP.1 MP.3 MP.5 MP.6	**Access Prior Learning**: In third grade, students used place value understanding to round whole numbers to the nearest 10 or 100. In Lesson 1-4, fourth grade students rounded whole numbers to place value positions. **Developing the Big Idea:** In this lesson, students will continue to use their place value understanding of rounding whole numbers to estimate sums or differences of whole numbers.	**Solve & Share:** The *Solve & Share* presents a *Put Together Take Apart Total Unknown* problem. Consider removing the bar diagram to see what tools, models or strategies students might use to solve the problem. Some students may find the actual sum for the problem. Choose students who may have estimated and another student who may have found the actual solution to share. Facilitate a discussion regarding the language of estimation (about, approximate, etc.). As well as, the purpose of estimation and that estimation helps mathematicians make sense of the problem. Guide students in understanding when estimation is appropriate and when to find the actual solution. **Visual Learning:** **Note:** Van de Walle, Karp and Bay-Williams (2010) state, "when several numbers are to be added, it is usually a good idea to round them to the same place value" (p. 246). In the *Visual Learning Animation*, students determine which estimate is correct. Ensure that students understand that either estimate is correct however, there may be an estimate that is closer or more reasonable to the actual answer. **Convince Me:** Use the *Convince Me!* to facilitate a discussion in regards to which estimate is more precise for the given situation. Be cautious in comments made in regards to who has a more precise estimate. Students need to understand that their estimation is not incorrect, but there may be a more precise estimate that is closer to the actual answer. ***-continues on next page-***

		Guided Practice: Consider doing items 1 and 2 of the *Guided Practice*. Have students explain their thinking in regards to why they estimated to a specific place value position. For item 2, ask students, "Which place value position would we round to, in order to have a more precise estimate?" **Independent Practice/Math Practices and Problem Solving:** Consider doing item 12 in the *Independent Practice* before having students complete *Quick Check* items (marked with a pink check mark). Have students discuss how they would estimate the sum. See item 12 "Coherence" for an explanation of this problem (TE, p. 55-56). **Assess and Differentiate/Intervention Activity:** In the *Intervention Activity*, place-value blocks are used, consider also having students use a number line to estimate.
Lesson 2-3: Add Whole Numbers		
4.NBT.B.4 **4.OA.A.3** MP.3 MP.7 MP.8	**Access Prior Learning**: In second and third grade, students added whole numbers with sums to 1,000. **Securing the Big Idea:** In this lesson, students will find the sum of multi-digit whole numbers by using the standard algorithm. Students will also estimate the sum by using various strategies.	**Note:** Encourage students to estimate sums of multi-digit whole numbers before finding the actual answer. This will help students develop reasonableness of numbers. Also, encourage students to use mathematical language like, addends and sum when discussing addition of whole numbers. **Solve & Share:** The *Solve & Share* presents an *Add To Result Unknown* problem. Consider having students use an estimation strategy before finding the actual sum. Look for students who use their understanding of place value and regrouping to find the sum. This would include partial sums. See example for partial sums for *Solve and Share* 4,219 + 3, 472 + 4, 436 4,000 + 200 + 10 +9 3,000 + 400 + 70 +2 + 4,000 + 400 + 30 +6 **Look Back:** In the *Look Back!,* students determine the property used to group numbers to add. In Lesson 2-1, the addition properties were discussed. **Visual Learning:** In the *Visual Learning,* students find the sum by using the U.S. Traditional standard algorithm. Consider having students use a standard algorithm and partial sums to compare strategies and support place-value understanding when using algorithms. Facilitate discussion around place value when using standard algorithms to add whole numbers. See questions in the *Prevent Misconceptions,* such as, Why do you need to align place values before adding? Explain how you regroup after adding the thousands. Why do you not regroup when adding the ten thousands? (TE, p. 60). Consider making an anchor chart with multiple addition strategies, like partial sums algorithm and the U.S. traditional algorithm. **Note**: During the *Visual Learning Animation*, the mathematical word ***variable*** appears. Consider using variables throughout the rest of this topic and beyond. Visual models, such as bar diagrams, are great representations to build conceptual understanding around this idea. See Teacher's Edition pages F31-F34 for various addition and subtraction situations with bar diagrams. **Independent Practice/Math Practices and Problem Solving:** Facilitate a discussion around students' estimation and reasonableness compared to their actual sums. **Assess and Differentiate/Intervention Activity:** Place-value charts are great tools to guide understanding and should include values related to the different place value positions. Consider also having students use the partial sums algorithm to compare to the U.S. traditional algorithm. ***CTC: *Solve & Share*** (student work samples)
Lesson 2-4: Subtract Whole Numbers		
4.NBT.B.4 **4.OA.A.3** MP.1 **MP.2** MP.3	**Access Prior Learning**: In third grade, students fluently subtracted whole numbers to 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationships between addition and subtraction (3.NBT.A.2).	**Note:** Karp, Bush and Dougherty (2014), state to be cautious with the phrase, "You cannot take a bigger number from a smaller number" as this is a rule that expires (p. 21). Meaning, this does not work when adding and subtracting negative numbers. A rule that stays true is "subtracting a larger number from a smaller one results in a negative number, an integer that is not in the set of whole numbers" (Karp, et al., 2014, p. 21). Keep focus on place value. See MP.3 Construct Arguments for example questions (TE, p.66). ***-continues on next page-***

	Developing the Big Idea: In this lesson, students will subtract greater whole numbers using the standard algorithm and place value.	**Solve & Share:** The *Solve & Share* presents a *Compare Difference Unknown* problem. Look for students who use place-value understanding; like partial differences algorithm to subtract whole numbers. Consider having these students share their strategy before having students who used the U.S. Traditional algorithm share. Facilitate a discussion around the similarities and differences between the strategies. Make a subtraction strategies anchor chart with strategies like the partial difference algorithm, U.S. traditional algorithm and other standard algorithms. **Visual Learning:** Connect student strategies from the *Solve & Share* to the *Visual Learning*. In the *Visual Learning Animation,* students discuss the inverse operation. **Convince Me:** Consider having students do the *Convince Me!* independently and then facilitate a discussion around the errors made. You may use this formatively to help guide the rest of the lesson. **Another Example:** Consider doing this activity whole class, because it discusses estimation when subtracting. **Independent Practice/Math Practice and Problem Solving** For item 16, students have to estimate to decide if their answer is reasonable. **Assess and Differentiate/Intervention Activity:** Consider doing the *Intervention Activity* with the whole class, so students see the minuend, 725, written as a different whole number using a place-value chart.
Lesson 2-5: Subtract Across Zeros		
4.NBT.B.4 **MP.2** MP.3 MP.5 MP.8	**Access Prior Learning**: In the previous lesson, students subtracted whole numbers using the standard algorithm and place value. **Securing the Big Idea:** In this lesson, students will continue subtracting greater multi-digit whole numbers using the standard algorithm and place value.	**Solve & Share:** The *Solve & Share* presents a *Compare Difference Unknown* problem. Students will be subtracting whole numbers across zeroes. Look for students who use place-value understanding; like partial differences algorithm to subtract whole numbers. Consider having these students share their strategy before having students who used the standard algorithm share. Facilitate a discussion around the similarities and differences between the strategies (Share and Compare). **Visual Learning:** Connect student strategies from *Solve & Share* to the *Visual Learning*. In the *Visual Learning*, students will learn two different regrouping strategies to subtract whole numbers across zeroes. Facilitate a discussion around specific questions such as, "Explain the regrouping in 6,000 – 4,678" (TE, p. 72). Have students use estimation to check if their answer is reasonable. **Independent Practice/Math Practice and Problem Solving:** Consider having students estimate as they work through the *Independent Practice* items as mentioned in the Think Bubble on p. 73. Item 26, discusses the mathematical terminology, variable from lesson 2-3, consider using this item to reinforce the use of variables. Also in the *Math Practices and Problem Solving* pages, item 28 "Higher Order Thinking" is a multi-step problem that uses information from a table to solve the problem. Consider using this formatively, as part of a whole-class discussion or during a Gallery Walk (Discussed in Topic 1 and ELL Toolkit). **Assess and Differentiate/Intervention Activity:** The *Intervention Activity* gives students an opportunity to use concrete manipulatives like place-value blocks or Teaching Tool 4 and 5 to solve subtraction problems across zeroes. This is a great place for all students to continue their understanding conceptually. ***CTC: *Quick Check*** (digital platform)
Lesson 2-6: Math Practices and Problem Solving- Reasoning		
4.NBT.B.4 **4.OA.A.3** **MP.2** MP.1 MP.4 MP.8	**Access Prior Learning**: Students have used Math Practice 2: Reasoning throughout the instructional materials and in previous grades. **Securing the Big Idea:** In this lesson, students will take previously learned concepts and skills related to addition and	**Solve & Share:** The *Solve & Share* presents a *Put Together Take Apart Addend Unknown* problem. Students will reason about whole number addition and subtraction to find sums and differences. Give students the opportunity to use a variety of tools, strategies or models to solve this problem. Consider having students "share and compare" student work of those who used **concrete** tools, drew a **representation** or solved **abstractly** by using an algorithm in this order. ***-continues on next page-***

	subtraction of whole numbers to reason abstractly and make sense of quantities and their relationships in problem situations.	**Look Back:** Use the *Look Back!* to encourage students to use bar diagrams to write numerical expressions and equations. Bar diagrams are the focus model in this lesson and will help students understand using variables to write numerical expressions and equations. **Visual Learning:** The *Visual Learning Animation* continues the use of bar diagrams and writing equations through reasoning about the problem. Discuss good mathematical thinking habits when reasoning, as well as using correct mathematical language when discussing. **Convince Me:** Have students do the *Convince Me!* as students are asked to create their own problem related to the bar diagram. Have students critique the reasoning of others to see if their problem works for the bar diagram given. **Assess and Differentiate/Intervention Activity:** If students are struggling with understanding bar diagrams, consider having students work on the *Intervention Activity*. A bar diagram can help students move from concrete tools to representational models. **Guided Practice:** Consider using items 1 and 2 whole class, as this lends itself to different problem situations. See Teacher's Edition pages F31-F34, page 88 in the 2010 Nevada Academic Content Standards or **Table 2: Common Addition and Subtraction Situations** below. If students are struggling with this problem, consider what they know and build upon their knowledge from there. **Independent Practice/Math Practice and Problem Solving:** To expose students to different problem situations, consider using items 3-5 or items 6-9. See Table 2 below for problem situations.

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	*A* bunnies sat on the grass. *B* more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	*A* bunnies were sitting on the grass. Some more bunnies hopped there. Then there were *C* bunnies. How many bunnies hopped over to the first *A* bunnies? $A + \square = C$	Some bunnies were sitting on the grass. *B* more bunnies hopped there. Then there were *C* bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	*C* apples were on the table. I ate *B* apples. How many apples are on the table now? $C - B = \square$	*C* apples were on the table. I ate some apples. Then there were *A* apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate *B* apples. Then there were *A* apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown[1]**	**Addend Unknown[2]**
Put Together /Take Apart	*A* red apples and *B* green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has *C* flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	*C* apples are on the table. *A* are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has *A* apples. Julie has *C* apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has *A* apples. Julie has *C* apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has *B* more apples than Lucy. Lucy has *A* apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has *B* fewer apples than Julie. Lucy has *A* apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has *B* fewer apples than Julie. Julie has *C* apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has *B* more apples than Lucy. Julie has *C* apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

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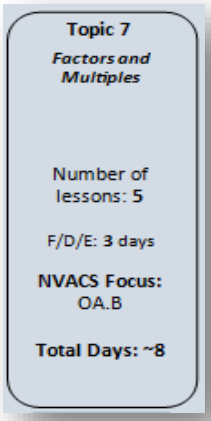
Grade 4 Topic 7: Factors and Multiples



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: Operations and Algebraic Thinking (pp. 29-31)
Prior to instruction, view the Topic 7 Professional Development Video located in Pearson Realize online. Read the Teacher Edition (TE): Cluster Overview/Math Background (pp. 365A-365F), the Topic Planner (pp. 365I-365J), all 5 lessons, and the Topic Assessments (pp. 405-406A).

Mathematical Background: Read Cluster Overview (TE, pp. 365A-365F)	**Topic Essential Questions:** How can you use arrays or multiplication to find the factors of a number? How can you identify prime and composite numbers? How can you find multiples of a number? *Reference TE p. 365 and Answering the Topic Essential Questions (TE, pp. 403-404) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

7-1	7-2	7-3	7-4	7-5	Assessment

3 F/D/E days used strategically throughout the topic.

Instructional note:
Topic 7 was moved before Topic 3 as it develops student understanding of factors and multiples by going back to accessing basic understanding of multiplication and division. The WCSD curriculum committee felt that accessing this understanding prior to developing understanding of multi-digit multiplication and division is foundational to support students' application of strategies (WCSD 4th Grade Curriculum Pacing Framework, p. 4).

The focus of this topic is on factors and multiples, as well as whether a whole number is prime or composite. Nevada Academic Content Standards (NVACS) 4.OA.B.4, state that students should begin to "gain familiarity with factors and multiples" (2010). Reys, Linquist, Lamdin, Smith, & Suydam state, "each of two numbers that are multiplied together to give a product is called a factor of that product…in looking at multiples, we usually begin with a number and generate multiples of the number" (2001, p. 288). Karen Karp states, "factors of a number can be found in pairs by thinking about multiplication…the product of any non-zero whole number and a given non-zero whole number is a multiple of both. Factors and multiples are closely related" (**enVision**math**2.0** PD Video). Multiples are products of any whole number.

Confusion for students may arise between factors and multiples. According to Reys, et al., the confusion happens "when teachers ask 'What is 36 a multiple of?' Teachers are actually asking children to find the factors of 36. Or 'What is 4 a factor of?' teachers are actually asking children to find the multiples of 4" (2001, p. 288).

Children need to see and understand the relationship between factors and multiples, and be able to think or move between the two concepts. Consider using a Venn diagram to facilitate this relationship between the two concepts. For example, 4 is a factor of 8 and 8 is a multiple of 4. Using a Venn diagram is also a useful organizer for students to begin to develop an understanding of common multiples between numbers. This commonality will support students when they begin to find common denominators when adding and subtracting unlike fractions in later grades.

One essential understanding about the relationship between factors and multiples is "whole numbers have a **distinct set** of whole number factors, but a whole number has an **infinite** number of whole number multiples" (Karen Karp, **enVision**math**2.0**, Topic 7 PD video).

Students will use their prior knowledge of factors to determine prime whole numbers and composite whole numbers. Prime numbers are whole numbers greater than one (1) that have exactly two factors; one and itself. Reys, et al., describe exploration that focuses on primes:

- twin primes (pairs of primes that are two apart, such as 11 and 13)
- reversal primes (pairs of primes, such as 79 and 97)
- or infinitude of primes
  (2001, p. 289).

This exploration will be important in Lesson 7-4 when students determine which numbers are prime or composite. Composite numbers are whole numbers greater than 1 with two or more factors. The emphasis on **whole numbers greater than 1**, for both prime and composite numbers will be imperative for students. Students will wonder about the numbers 0 and 1 being prime or composite, but the emphasis on **whole numbers greater than one** will help students understand that **0 and 1 are neither prime nor composite**.

Focus Math Practice 8: **Repeated reasoning**
Focus opportunities for students to develop *Mathematical Practice 8* behaviors as this is the focus of the Math Practices and Problem Solving, lesson 7-3. Reference the Teacher's Edition (pp. F28-F28A) and the NVACS (2010, p. 8).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
factor pairs multiples generalize prime number composite	*factor*

Additional terminology that students may need support with: arrays, arrange, groups of, rows, columns, repeated factor pair, Commutative Property of Multiplication

*Consider using the additional terminology to label anchor charts used throughout this topic.

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "What tools, strategies or models are students using to find all factors of a whole number?"
"What tools, strategies or models are students using to find multiples of a whole number?"

Lesson	Evidence	Look for
7-3	***Independent Practice*** (digital platform or student work samples) Items 3, 4 and 5	Focus CTC on the big idea: • students recognize and generate patterns in factoring whole numbers.
7-5	***Quick Check*** *(*digital platform*)*	Focus CTC on the big idea: • students recognize and generate patterns in finding an infinite amount of multiples.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 403-406	Use *Scoring Guide* TE pp. 403-406A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 7-1: Understand Factors		
4.OA.B.4 MP.2 MP.3 MP.5 MP.7	**Access Prior Learning**: In third grade (3.OA.A.1), students learned to interpret products of whole numbers as the total number of objects in *n* groups of *n* objects. Students also learned to use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, etc. (3.OA.A.3). **Developing the Big Idea:** Students will continue to develop their understanding of multiplication by finding factor pairs for whole numbers in the range of 1-100.	**Solve & Share:** Consider giving students the opportunity to use tools other than graph paper. Students may use 2-colored counters, place-value blocks or other tools that may be put into rows and columns using 24 total objects. **Visual Learning:** In the video, students find all the factors for 12. Consider having students use tools during the video to find the factor pairs for 12, or have students determine the factor pairs for 12 before showing the video. **Guided Practice:** Consider using the *Guided Practice* items 1 & 2 whole group, as factors are discussed as the dimensions of a rectangular array. These items continue to develop an understanding of area. Also, consider doing item 4 whole group as it addresses square numbers. See item 4 at the bottom of the Teacher's Edition (pp. 371-372) ***-continues on next page-***

		Independent Practice/Math Practices and Problem Solving: After students work on *Quick Check* items (pink check marks), facilitate a discussion around item 15 whole class. This item continues the focus on factors and begins to develop an understanding of divisibility (when a number ends in a 0 or 5, it can divide by 5 evenly or 5 is a factor of that number). **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with students who may need extra support with finding factors of numbers. Continue to encourage the use of tools for students or use Teaching Tools 9 and/or 16.
Lesson 7-2: Factors		
4.OA.B.4 MP.1 MP.2 MP.3 MP.4 MP.5	**Access Prior Learning**: In the previous lesson, students learned the meaning of factors and how to find factors of a number. **Developing the Big Idea:** In this lesson, students will continue their work with factors and factor pairs.	**Solve & Share:** Consider giving students an opportunity to use multiple tools or representations to complete the *Solve & Share*. Consider removing the grid, to enable students to use tools. Suggested tools include two-colored counters, place-value blocks, grid paper or other manipulatives with 20 objects. When students find factor pairs for 20, they may have more than 3 ways. Some students may have 4 by 5 and a 5 by 4. When having students to share their answers, have students share who may have more than 3. Ask students, "Does a 4 by 5 rectangle and a 5 by 4 rectangle have the same area?" You may consider discussing the **Commutative Property of Multiplication** (students learned the Commutative Property in third grade). The Commutative Property of Multiplication will be discussed further in Topic 3. **Look Back:** In the *Look Back!*, students determine how they know if they have found all factor pairs. Consider having a whole group discussion around this idea. **Visual Learning:** Factor pairs and multiples are discussed in the *Visual Learning Animation*. Connect these mathematical terms to what students did in the *Solve & Share*. **Convince Me:** Consider using the *Convince Me!* as part of the formative assessment process to see if students understand how to find factor pairs and when they know all factor pairs have been found. If students have an understanding of this concept, consider having students complete selected *Independent Practice/Math Practices and Problem Solving* items. If students are still struggling, consider using the *Guided Practice* items with these students. **Guided Practice:** Consider using item 2 as a whole class discussion, as students are asked to find a factor in all even numbers. This item addresses some divisibility understanding for students (all even numbers are divisible by 2). Watch for students who invent this concept on their own. **Independent Practice/Math Practices and Problem Solving:** Consider using item 26 whole group, as students solve a multiplicative comparison problem with "3 times" as many. Multiplicative comparison will be further discussed in Topic 6, but this item will give students some exposure to this mathematical idea. **Assess and Differentiate/Intervention Activity:** If students are still struggling with finding factor pairs after two lessons on factors, consider using the *Intervention Activity* or *Reteach* page with students. Again, encourage tool use. Consider utilizing the following question formats during practice: Example 1 Full Statement **Example Stem:** Which list has **all** of the factor pairs of 36? A. 1 × 36, 2 × 18, 3 × 12, 4 × 9, 6 × 6 B. 1 × 36, 2 × 16, 3 × 12, 4 × 9, 6 × 8 C. 1 × 36, 2 × 18, 3 × 6, 4 × 8, 6 × 12 D. 1 × 36, 2 × 12, 3 × 9, 4 × 8, 6 × 6 ***-continues on next page-***

		Example 1 Full Statement **Example Stem:** Which numbers are factors of **both** 18 and 45? A. 1, 2, 8 B. 1, 3, 9 C. 1, 4, 8 D. 1, 5, 9
Lesson 7-3: Math Practices and Problem Solving- Repeated Reasoning		
4.OA.B.4 **4.NBT.B.5** **MP.8** MP.1 MP.2 MP.3 MP.4 MP.6	**Access Prior Learning**: In previous lessons, students have worked with factors, factor pairs and multiples. Students have also worked with repeated reasoning in previous grades, topics and lessons. **Developing the Big Idea:** In this lesson, students will continue working with factors, factor pairs and multiples by making generalizations from repeated reasoning.	**Note:** Most *Math Practices and Problem Solving* lessons are at the end of each topic, but in this topic the *Math Practice and Problem Solving* lesson is in the middle of the topic. Students will develop a generalization when all factor pairs start repeating, all factors have been found for that number. **Solve & Share:** Consider removing the table with the example and non-example of a rectangular array to formally assess students' prior knowledge with arrays. Give students an opportunity to use concrete tools to arrange 24 objects into a rectangular array. Look for students who may have used multiplication or division to find all the factors and factor pairs for 24. Look for students who may have connected the commutative property to the array arrangement. Students may not recall the name of the property, but may have examples of the commutative property. Consider choosing these students to share. **Visual Learning:** The *Visual Learning Animation* has students making generalizations related to factor pairs and factors. Consider making an anchor chart with ways students can generalize repeated reasoning. **Convince Me:** The *Convince Me!* has students use a diagram to justify that all factor pairs have been found for the number 24. Consider having students come up with their own way to ensure all factor pairs have been found for 24. Students can generalize all factor pairs have been found when they begin to see a reversal in the pairs. Consider using the *Convince Me!* formatively to assess student understanding. If students seem to have an understanding of factors and factor pairs, have students begin the *Guided Practice* and *Independent Practice/Math Practice and Problem Solving* on their own. Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** Which factor of 24 is missing in this list of numbers? 1, 2, 3, 4, 6, __, 12, 24 ***CTC:** ***Independent Practice items 3-5*** (digital platform or student work samples)
Lesson 7-4: Prime and Composite Numbers		
4.OA.B.4 **MP.8** MP.2 MP.3 MP.7	**Access Prior Learning**: Students worked with factors and factor pairs in lessons 7-1, 7-2 and 7-3. **Beginning the Big Idea:** In this lesson, students will use their understanding of factors and factor pairs to decide which numbers are prime or composite based on how many factors or factor pairs make up the number.	**Solve & Share:** The idea of the *Solve & Share* is for students to use each color individually. For example, students will use the red color tiles to make arrays of 2 x 1, 1 x 2. Consider using an anchor chart as students share all the possible arrays (factors and factor pairs) and label the different arrays. In the example, the red color tiles are labeled 2 x 1, 1 x 2. Support students in connecting the reasoning that some of the colored tiles have only two arrays, while some colored tiles have multiple arrays. This begins to support the mathematical concept of prime and composite whole numbers. **Visual Learning:** *Prevent Misconceptions* addresses the numbers 0 and 1 (TE, p. 388). Zero (0) and 1 are neither prime nor composite. You may consider having students determine if these numbers are prime or composite after the discussion around the *Visual Learning* has taken place. ***-continues on next page-***

		Students need to understand that a prime number is a **whole number greater than 1** that has exactly two factors, 1 and itself. A composite number is a **whole number greater than 1** that has more than 2 factors. This discussion will help students on item 25 of *Math Practices and Problem Solving*, which is a *Quick Check* item. **Convince Me:** Consider using the *Convince Me!* formatively to determine if students have developed an understanding of prime and composite numbers. Do students understand that prime and composite numbers are different? Are they able to explain why a number cannot be prime nor composite? **Independent Practice/Math Practices and Problem Solving:** Items 20 and 21 in *Math Practices and Problem Solving* ask students to use the graph to solve these items. As a consideration, ask students to solve and then discuss as a whole group how to use information in a graph to solve problems. Consider using item 17 of the *Homework & Practice* pages during class instruction. You may consider having students work in small groups or partners on this activity. Prompt and support students to discover the patterns on the Number Chart for prime and composite numbers. Discuss whole group after students have had some opportunity to discover any patterns. Consider providing students a pre-printed number chart to 100 if time is a concern. **Assess and Differentiate/Intervention Activity:** Consider using the *On-Level* and *Advanced Center Games* with all students. This game may be played multiple times, as there is a range of numbers that may be used with each of the criteria given on the game board. Consider utilizing item 8 from the *Topic Assessment* (pp. 404) in a whole group discussion during the lesson. Item 8 asks students to choose all true statements. This is a great opportunity for students to think about the characteristics of prime and composite whole numbers. Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** Decide whether each number is prime or composite. Click in the table to respond. | | Prime | Composite | | --- | --- | --- | | 17 | | | | 52 | | | | 87 | | | Consider using item 8 from the *Topic Assessment* (pp. 404) in a whole group discussion during the lesson.
Lesson 7-5: Multiples		
4.OA.B.4 **MP.8** MP.2 MP.3 MP.4 MP.7	**Access Prior Learning**: Students learned about factors and factor pairs in lessons 7-1, 7-2 and 7-3. In third grade, students also learned a product is the solution to a multiplication problem. **Developing the Big Idea:** Students will continue to build their understanding of factors and the relationship between a factor and a multiple or product.	**Solve & Share:** Consider removing the table on the *Solve & Share* to enable students to derive strategies that reflect higher-level thinking or increased cognitive demand. Watch for students who may see a pattern and the relationship between golf balls per player and golf balls in play as they work through the problem. Students construct the understanding that the relationship is a multiplication problem. Consider supporting students, they only derive 9 x 1, with guiding questions such as, "How many golf balls would be in play if each player had 3 golf balls?" **Visual Learning:** Students will "extend the idea of decomposition to multiplication and learn to use the term "multiple" (CCSWT, 2011). **Convince Me:** The *Convince Me!* addresses the context of a problem. Students need to develop the understanding of when it is not possible to go to the next multiple, as that would be greater than 60. Students might want to continue with finding many multiples of 8; if so, address the context of the problem. This will develop students' problem solving skills. ***-continues on next page-***

Guided Practice:
Items 1 and 2 continues the work with the Ferris Wheel problem. These items may help students determine why a multiple does not work for a particular number. Students may use their understanding of factors to support their work with multiples.

Independent Practice/Math Practice and Problem Solving:
Item 29 uses a Venn diagram to organize common multiples for numbers. Consider using this item whole group. By doing this problem together, students begin to see numbers have common multiples and this knowledge will support them when they begin to find common denominators when adding and subtracting fractions with unlike denominators.

Assess and Differentiate/Intervention Activity:
Reteach and *Homework & Practice* uses a multiplication table to find multiples. What is the purpose of the multiplication table? Is it a tool? Or used procedurally?

As students find multiples, have students pay close attention to the arrays that are formed. For example, 49 is a multiple of 7. A 7 by 7 array has 49 squared units. Emphasize the use of the table as a tool and avoid making it into a procedural process for students.

Consider utilizing this question format during practice:

Example 1
Full Statement

Example Stem: Decide whether each number is a multiple of 6, a factor of 6, or neither. Each number may be matched to more than one description. Select the correct response in the table.

	Multiple of 6	Factor of 6	Neither a Multiple Nor a Factor of 6
1			
2			
3			
6			
8			
12			

CTC:** ***Quick Check (digital platform)

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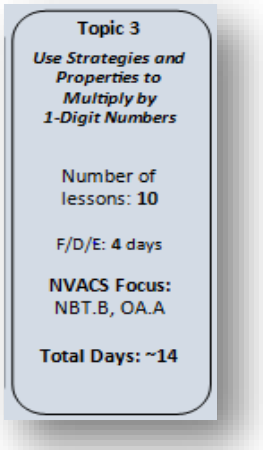
▸ Grade 4 Topic 3: Use Strategies and Properties to Multiply by 1-Digit Numbers



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: Number and Operations in Base Ten (pp. 12-15)

Prior to instruction, view the Topic 3 Professional Development Video (located in Pearson Realize online). Read the Teacher Edition (TE), Cluster Overview/Math Background (pp. 43A-43F), the Topic Planner (pp. 91I-91J), all 10 lessons, and the Topic Assessments (pp. 165-166A).

Mathematical Background:	**Topic Essential Questions:**
Read Cluster Overview (TE, pp. 43A-43F)	How can you multiply by multiples of 10, 100 and 1,000? How can you estimate when you multiply? *Reference TE page 91 and Answering the Topic Essential Questions (TE, pp. 161-162) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

3-1	3-2	3-3	3-4	3-5	3-6	3-7	3-8	3-9	3-10	Assessment

4 F/D/E days used strategically throughout the topic.

Instructional note:

This topic focuses on using place-value understanding and estimation to multiply multi-digit whole numbers. Focus instruction on Nevada Academic Content Standards (NVACS) standards 4.NBT.B.5. Emphasis for standard 4.NBT.B.5 is to "multiply a whole number of up to four digits by a one-digit whole number using strategies based on place value and the properties of operations".

The problem types or structures in this topic focuses on equal groups and multiplicative comparison when multiplying. Van de Walle, Karp, Bay-Williams state that equal group problems can be repeated addition or rate problems. "Repeated addition problems can be problems such as, 'If three children have four apples, how many apples are there?" and rate problems may be, "If there are four apples per child, how many apples would three children have?'" (2010, pp. 154-155). Multiplicative comparison problems are when there are two different sets and can be more complex for developing students mathematical understanding of multiplication. Van de Walle, et al., state, "One set consists of multiple copies of the other. An example of a multiplicative comparison problem is, 'Jill picked 6 apples. Mark picked 4 times as many apples as Jill. How many apples did Mark pick?'" (2010, p. 155).

The distributive, commutative and associative properties of multiplication, as well as the partial product algorithm using an area model or open array are all areas of focus in this topic. Additionally, this topic does show students the U.S. standard algorithm. The U.S. standard algorithm for multiplication is based on the distributive property. It is a very efficient procedure, yet also one of the most difficult algorithms for students to understand if taught in a systematic procedural manner. Instruction that includes the conceptual knowledge behind procedures is crucial. Research shows that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Van de Walle, et al., (2010) states, "once having begun with traditional algorithms, it is extremely difficult to suggest to students that they learn other methods" (p. 217).

Be cautious introducing the U.S standard algorithm without conceptual understanding, as students are introduced to multi-digit multiplication of whole numbers. Use of area models and the partial products algorithm rely on the distributive property and can be used to help students develop the conceptual understanding necessary to become fluent with multi-digit multiplication. These algorithms allow students to see how the partial products are created while avoiding the errors that often occur when regrouping and recording using the U.S. standard algorithm. They also allow students to work horizontally or vertically, and to multiply factors and add partial products in varying order. You may consider using partial products algorithm instead of the U.S. standard algorithm, especially until students are secure with the understandings of how area models and partial products support the distributive property leading to more algorithmic work.

Area models and the partial products algorithm can be very efficient. Using these strategies, students create separate values for each partial product without much more additional time or writing when compared to the standard algorithm. The area model has many advantages over the standard algorithm, especially as students begin using two-digit multipliers (Van de Walle, Karp, Lovin, & Bay-Williams, 2014). Students should understand that multiple strategies can be used to solve these problems with some being more appropriate than others in certain instances. Effective instruction provides experiences that help students connect procedures with the underlying concepts (NCTM, 2014).

Estimation is important as students use these strategies to check for reasonableness of their answers. In this topic students will also compare and round whole numbers. Rounding whole numbers is an **estimation** strategy. "The term estimation refers to a number that is a suitable approximation for an exact number given the particular context" (Van de Walle, et al., 2010, p. 241). Rounding is one strategy used to estimate. Number lines are useful tools to help students round numbers. Other estimation strategies include; compatible numbers, front-end methods, clustering and using tens and hundreds. Students should be able to use and recognize words and phrases for estimation like; about, approximately, close to; etc. Van de Walle, et al. write,

> Do not reward or emphasize the answer that is the closest. It is already very difficult for students to handle "approximate" answers; worrying about accuracy and pushing for the closest answers only exacerbates this problem. Instead, focus on whether the answers given are *reasonable* for the situation or problem at hand. (2010, p. 242)

Students should be able to use rounding flexibly and understand it conceptually, so it can be a useful estimation strategy.

Compatible numbers are "two or three numbers that can be grouped to make benchmark values. If the numbers in the list can be adjusted slightly to produce these groups, that will make finding an estimate easier" (Van de Walle, et al., 2010, p. 247). Karen Karp, Sarah Bush and Barbara Dougherty, write in "13 Rules that Expire", that the zero trick or teaching children to multiply a number by ten by just adding a zero to the end of the number will break down or "expire" in student learning trajectories. The "zero trick" expires when students begin to multiply decimals in 5th grade and this rule no longer applies. Be cautious when teaching the "zero trick" and consider avoiding it all together. Instead, consider connecting it to our place-value base-ten system discussed in Topic 1.

"The equal sign is one of the most important symbols in elementary arithmetic, in algebra, and in all mathematics using numbers and operations" (Van de Walle, et al., 2010, p. 258). It is important that students understand the meaning of the equal sign and not simply view it as the answer to a problem. Understanding the equal sign means "to see, understand and symbolize the relationships in our number system. When students fail to understand the equal sign, they typically have difficulty when it is encountered in algebraic expressions" (Van de Walle, et al., 2010, p. 258).

Focus Math Practice 4: **Model with math**
Focus on opportunities for students to develop *Mathematical Practice 4* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 3-10. Reference the Teacher's Edition (pp. F24-F24A) and the NVACS (2010, p. 7).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
numerical expression partial products	*Associative Property of multiplication* *Distributive property* *compensation* *Commutative Property of multiplication*

Additional terminology that students may need support with: area model, array or open arrays, estimate, rounding, factors, product, mental math, reasonableness, overestimate, underestimate

*Consider using the additional terminology to label anchor charts used throughout this topic.

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students developing conceptual understanding and moving their thinking from less sophisticated understandings (place value blocks) toward equations through use of repeated addition or multiplication using the partial products algorithm?"

Lesson	Evidence	Look for
3-2	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • student strategies and models. • student estimation; overestimation or underestimation related to actual answer.
3-6	***Convince Me!*** (digital platform or student work samples)	Focus CTC on the big idea: • partial products allows for students to start in any place value. • connects to the distributive property. Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 161-166	Use *Scoring Guide* TE pp. 161-166A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 3-1: Mental Math- Multiply Multiples of 10, 100 and 1,000		
4 NBT.B.5 MP.2 **MP.4** MP.7	**Access Prior Learning**: In third grade, students multiplied whole numbers by multiples of 10. In Topic 1, Students also learned about the base 10 system and 10 times. **Developing the Big Idea:** In this lesson, students will use their previous knowledge and multiply by 10, 100 and 1,000 using mental math.	**Solve & Share:** Consider giving students opportunity to use multiple tools or representations to complete the *Solve & Share*. **Look Back:** Consider having students include the *Look Back!* question as they answer the *Solve & Share*. Look for students who make connections to the work they did in Topic 1 around the 10 times idea. **Visual Learning:** In the *Visual Learning Animation*, students will make connections in multiplication by using the associative property of multiplication. Consider making an anchor chart with the associative property. Add the other properties of multiplication as they are discussed in this topic. Consider pausing the animation to have an in-depth conversation regarding the property and why the "short-cut or zero trick" works. The "zero trick" works, because it connects to shifting numbers within the place-value structure. See the instructional note regarding the "zero trick". **Note:** Consider moving to the *Intervention Activity* after the *Visual Learning Animation*. **Another Example:** Consider using the *Another Example!* as it discusses place-value relationships and patterns when multiplying by 10, 100 and 1,000. Consider using *Another Example!* before the *Convince Me!*. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students as it uses concrete tools to explain multiplying by multiples of 10, 100, or 1,000. Some students need concrete or representational models before an abstract algorithm. Consider using the *Intervention Activity* before the *Convince Me!* and after *Another Example!*.
Lesson 3-2: Mental Math- Round to Estimate Products		
4.NBT.B.5 MP.2	**Access Prior Learning**: In previous Topics 1 and 2, students learned how to estimate by rounding whole numbers, sums and differences.	**Solve & Share:** Due to estimation lessons in previous topics, look for students who observed the word "about" in the *Solve & Share*. Consider sharing strategies where students did estimate to solve for the product. ***-continues on next page-***

	Developing the Big Idea: In this lesson, students will estimate products of multi-digit whole numbers to check if they are reasonable.	**Look Back:** Consider facilitating a discussion around the *Look Back!*, as students begin to think about estimation as an underestimate (less than the actual number) or overestimate (greater than the actual number) based on how they chose to estimate. **Convince Me:** Rounding is used to estimate each class' money earned for items sold. Remember, rounding is an estimation strategy. Consider adding *underestimate* and *overestimate* to your estimation anchor chart after discussion around the *Convince Me!***.** **Another Example:** In the *Another Example!*, students discuss how an estimate is reasonable. Consider using this example with all students, so students continue to reason through estimation. Students should use estimation throughout the lessons in this topic. **Independent Practice/Math Practices and Problem Solving:** Items 12 and 13 deal with multiplicative comparison problems. Consider guiding students in a whole or small group discussion if assigning these items. **Assess and Differentiate/Intervention Activity:** If students are still struggling with estimation, consider using the *Intervention Activity* to scaffold understanding. The *Intervention Activity* uses a number line to help students round numbers based on their proximity to landmarks. ***CTC**: ***Solve & Share*** (student work samples)
Lesson 3-3: The Distributive Property		
4.NBT.B.5 MP.1 MP.2 **MP.4** MP.7	**Access Prior Learning**: In third grade, students learned the distributive property and used the property to understand strategies for remembering basic facts. **Developing the Big Idea:** In this lesson, students will review the distributive property and use it to find products for multi-digit whole numbers.	**Note:** Consider reading the instructional note for item 12 before teaching the lesson (TE, pp. 109-110). **Solve & Share:** Students learned to determine area in third grade. Students will use this knowledge of area to answer the *Solve & Share* looking for the unshaded area. The *Solve & Share* begins to have students think about the distributive property with subtraction. **Look Back:** Consider using the *Look Back!* after the *Solve & Share*. Watch for students who do not see equality in the two equations. This may be a place to consider discussing the true meaning of the equal sign. The equal sign does not mean "the answer to", it means "the same as" or "equivalent to". Have students think about the equal sign as a balance. See instructional note for more information regarding the equal sign. **Visual Learning:** Mathematical language of numerical expression and distributive property is discussed in the *Visual Learning Animation*. Consider having students estimate the product before solving the expression. In the *Visual Learning Animation*, students use place-value understanding to solve the expression. Consider having a discussion around the importance of place-value while multiplying, as well as how an area model or open array supports place-value. **Convince Me:** Refer to the note for the *Convince Me!* in the TE (pg. 108) to guide the conversation. **Another Example:** Consider using the *Another Example!* with the whole class. The *Another Example!* discusses the distributive property with addition and the distributive property of subtraction. Consider using this as part of a classroom discussion to elicit student invented strategies. Students may invent the distributive property with addition or subtraction. If students do not come up with one or the other, consider using a past student's name with the "invented strategy". **Guided Practice:** Consider using item 1 with the whole class as the area model and distributive property are used simultaneously. Consider having students figure out the equation. For example, students are finding the product for 4 X 13 and there are two ways to find the product. Ask students to find other ways to determine the product. For example, students may say, "4 x (5 + 8) = (4 X 5) + (4 X 8)". Make an anchor chart with student responses. **Independent Practice/Math Practices and Problem Solving:** For items 3-10, consider having students estimate before using the distributive property to solve in order to check the reasonableness of their answer. ***-continues on next page-***

		Many of the problems in the *Math Practices and Problem Solving* ask students to use the distributive property with addition or subtraction. Consider using item 12 with students before independent work. Refer to the Coherence Note in the TE (pg. 109-110) for more information.
Lesson 3-4: Mental Math Strategies for Multiplication		
4.NBT.B.5 MP.2 MP.3 **MP.4** MP.7	**Access Prior Learning**: In the third grade, students used the distributive property as a strategy for finding basic facts. Students also used the Associative and Commutative Property to multiply. In the previous lessons, students have used the distributive property and mental math to multiply numbers by 10, 100, 1000. **Developing the Big Idea:** In this lesson, students will use the distributive, associative and commutative properties of multiplication to find products for multi-digit whole numbers mentally.	**Solve & Share:** Consider using the *Solve & Share* as a Number Talk. A Number Talk is where students solve each of the problems mentally and explain their thinking. Record students' responses on the board or chart paper. Students may use the Associative or Commutative Property of Multiplication to solve the problem. For example, the problem 25 x 9 x 4, students may solve the problem by using the commutative property and compatible numbers. Students may multiply 25 and 4 to get 100 and then multiply 100 and 9 to get 900. **Look Back:** Consider having students work on the *Look Back!* on their own, then discuss different ways students may have solved the problem whole group. **Visual Learning:** The *Visual Learning Animation* discusses the commutative and associative property of multiplication. Consider using the students' responses from the *Solve & Share* to reiterate these properties and compare to the animation. **Another Example:** The focus of the *Another Example!* is compensation. Compensation is an estimation strategy. Students use the information from the *Visual Learning* to solve the multiplication problem. Consider using this as a whole class Number Talk to see if a student uses compensation to solve the problem. If a student does not use compensation, consider using a student's name from a previous year and share the compensation estimation strategy with your class. **Independent Practice/Math Practices and Problem Solving:** Item 13 is a multi-step addition and subtraction problem, with the start unknown. Consider using this item to review Topic 2 or for summative grading purposes.
Lesson 3-5: Arrays and Partial Products		
4.OA.B.4 **MP.8** MP.2 MP.3 MP.4 MP.7	**Access Prior Learning**: In third grade, students used place-value blocks and other tools to make arrays. **Developing the Big Idea:** In this lesson, students will use place-value blocks and other tools or models to find partial products to multiply a 1-digit whole number by a 3-digit whole number.	**Solve & Share:** Consider having students estimate before solving the problem. Consider sharing students' strategies who may have used place-value blocks and the partial product algorithm to solve the problem. Both of these student strategies will be discussed in the *Visual Learning Animation*. **Visual Learning:** The partial products algorithm is discussed in the animation. The partial product algorithm uses place-value understanding to multiply multi-digit whole numbers. Connect the distributive property from previous lessons to the partial product algorithm. Also, note the place-value blocks are a representation of an array. Place-value blocks are used as an array throughout the lesson whether concrete or representational. **Convince Me:** Ask students to connect the partial products algorithm to place-value blocks (concrete). Have students think about how the place-value blocks can be shown representationally. Having students draw the place-value blocks will connect to the *Another Example!*, which has students draw a model. **Another Example:** Consider using the *Another Example!* as students use representations to 1,000 as well as partial products to solve the problem. **Guided Practice:** Consider giving students opportunities to use tools or other representations as they solve items 1-2 using the partial product algorithm to compare. **Independent Practice/Math Practice and Problem Solving** Consider having students complete item 8 and then discuss whole class. In item 8, students decide what multiplication equation is represented by the place-value blocks. Remind them of the problem in the *Visual Learning Animation* to help guide them with the problem. For item 9, consider having students estimate and discuss overestimation and under-estimation.

Lesson 3-6: Use Partial Products to Multiply By 1-Digit Numbers		
4.NBT.B.5 **MP.4** MP.5 MP.7 MP.8	**Access Prior Learning**: In the previous lesson, students used an array of place-value blocks to find partial products to multiply a 1-digit whole number by a 3- or 4-digit whole number. **Developing the Big Idea:** In this lesson, students will continue using the partial product algorithm to multiply.	**Note:** This lesson is similar to Lesson 3-5. Students continue developing their understanding of the partial product algorithm. **Solve & Share:** Consider having students estimate before solving the problem. Also, consider prompting students to use more than one strategy to answer the *Solve & Share* problem. **Convince Me:** Read the notes at the bottom of the Teacher's Edition (p. 126) before teaching the lesson. Consider using the *Convince Me!* with all students as they develop an understanding that they can start with any place-value position when using the partial products algorithm. **Independent Practice/Math Practices and Problem Solving:** For items 4-7, consider having students estimate before finding the actual answer in order to check the reasonableness of their answer. Item 16 is a *Quick Check* item and a multiplicative comparison problem. Consider using this formatively to assess student understanding, with an emphasis on context within the student's explanation. **Assess and Differentiate/Intervention Activity:** Consider using the questions to facilitate a discussion around the *Intervention Activity* as the activity has students thinking about generalizing place-value understanding when using the partial product algorithm. Consider giving students an opportunity to use place-value blocks or other representations as they solve using the partial product algorithm. ***CTC: *Convince Me!*** (digital platform or student work samples)
Lesson 3-7: Multiply 2- and 3-Digit Numbers by 1-Digit Numbers		
4.NBT.B.5 **4.OA.A.3** MP.1 MP.2 MP.3 **MP.4** MP.8	**Access Prior Learning**: In the previous lessons, students learned and applied the partial product algorithm. **Beginning of the Big Idea:** In this lesson, students will begin to learn the U.S. standard algorithm for multiplication.	**Solve & Share:** Consider removing "How can you use paper and pencil to find how many books were ordered?" Ask students to estimate and solve the problem any way they choose, even if it is using place-value blocks. **Visual Learning:** In the *Visual Learning Animation*, students are shown the U.S. traditional algorithm for multiplication. Consider showing or not showing students the algorithm. Do not expect students to be secure in the U.S. traditional algorithm this year, as students will have multiple opportunities in the future to become secure. Consider having students compare the partial product algorithm and the U.S. traditional algorithm. See the *Prevent Misconceptions* for how and where students struggle with U.S. traditional algorithm (TE, p. 132). **Convince Me:** Consider encouraging students to use multiple strategies or models to solve this problem. Consider recording strategies on an anchor chart to show the comparison. Continue having students estimate to check for reasonableness. **Another Example:** In the *Another Example!*, students continue comparing the partial products algorithm to the U.S. traditional algorithm. Consider recording strategies on an anchor chart to show the comparison. Continue having students estimate to check for reasonableness. **Independent Practice/Math Practices and Problem Solving:** Items 15-18 are multiplicative comparison problems. Consider discussing the context of the problems with students and encouraging use of the bar diagram. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students as it brings in the concrete tools to explain the U.S. traditional algorithm more in-depth by using place-value understanding and names to explain the algorithm. It also demonstrates how regrouping works.
Lesson 3-8: Multiply 4-Digit by 1-Digit Numbers		
4.NBT.B.5 **4.OA.A.3** MP.2 MP.4	**Access Prior Learning**: In the previous lesson, students continued multiplying numbers by using partial products algorithm and the U.S. standard algorithm.	**Solve & Share:** Consider accepting multiple strategies or models students used to solve problem. Consider having students do a Gallery Walk (ELL Toolkit, p. 22) as well as critique the reasoning of others. ***-continues on next page-***

MP.8	**Beginning of the Big Idea:** In this lesson, students will continue to use algorithms to multiply whole numbers.	**Visual Learning:** Consider not showing the *Visual Learning Animation* as it shows the U.S. traditional algorithm as a Step-by-Step procedure. Continue to give students the opportunity to use the partial product algorithm as it addresses multiplication through place-value understanding. **Convince Me:** Consider facilitating a discussion around the *Convince Me!* as it reinforces estimation. **Independent Practice/Math Practice and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider using item 26 with whole class, as it asks students to use a bar diagram to solve the problem. This may help support students who are struggling with multiplication and an algorithm. Students see multiplication as repeated addition. Items 18, 23, and 25 involve multi-digit subtraction, so consider using this time for review or for a grade. **Assess and Differentiate/Intervention Activity:** Consider using the partial product algorithm or arrays to provide additional support for students instead of color-coding place-value in the U.S. traditional algorithm.
Lesson 3-9: Multiply By 1-Digit Numbers		
4.NBT.B.5 **4.OA.A.3** MP.2 MP.3 MP.6 MP.8	**Access Prior Learning**: In Lessons 3-7 and 3-8, students compared the partial product algorithm to the U.S standard algorithm. **Developing the Big Idea:** In this lesson, students will continue using the algorithms to multiply whole numbers.	**Note:** Consider having students continue to estimate to check for reasonableness as they progress through the lesson. **Solve & Share:** Consider sharing and comparing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. **Look Back:** Consider using the *Look Back!* as students may use their understanding of doubling to answer the problem. Share a student's response who may have used doubling or introduce doubling as a past student's strategy. **Visual Learning:** Estimation is an important understanding to be developed. Ask students to use estimation as an ongoing strategy to support building place-value understanding. Consider giving students the opportunity to use any strategy or model to solve the problems in the *Visual Learning Animation*. Consider using the first two problems as a Number Talk and have students share how they may have solved the problems mentally. **Independent Practice/Math Practice and Problem Solving:** Item 31 is a *Quick Check* item and a multi-step multiplicative comparison problem. Consider asking students to explain or model their thinking to assess conceptual understanding of the problem. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students, as this activity emphasizes place-value understanding when multiplying and regrouping.
Lesson 3-10: Math Practices and Problem Solving- Model With Math		
4.NBT.B.5 **MP.4** MP.1 MP.2 MP.5 MP.6	**Access Prior Learning**: In third grade, students modeled and solved addition, subtraction and basic fact multiplication problems by drawing bar diagrams and writing equations. **Developing the Big Idea:** Students will continue to model with math using bar diagrams as they multiply multi-digit whole numbers.	**Solve & Share:** The *Solve & Share* is a multiplicative comparison problem. Consider using a bar diagram, but ask students to solve the problem in a different way. For more information regarding multiplication and division problem types, please see the Teacher's Edition (pp. F35-F36) or the Nevada Academic Content Standards (2010, p. 89). **Convince Me:** Consider facilitating a discussion around the *Convince Me!* as students use derived facts (known facts) to help them estimate and decide if their answer makes sense. **Guided Practice:** *Guided Practice* items 1-3 are multiplicative comparison problems and are different then the *Visual Learning Animation*, which is a grouping problem. Consider using the *Guided Practice* whole group and compare item 1 to the *Visual Learning* to see if students can see the difference between the two problem types. ***-continues on next page-***

		Consider utilizing this question: Example 2 Full Statement **Example Stem: 1:** Enter the unknown number that makes the equation true. 120 × 5 + □ = 603

Table 3: Multiplication and division situations

	A × B = □	A × □ = C and C ÷ A = □	□ × B = C and C ÷ B = □
Equal Groups of Objects	Unknown Product There are A bags with B plums in each bag. How many plums are there in all?	Group Size Unknown If C plums are shared equally into A bags, then how many plums will be in each bag?	Number of Groups Unknown If C plums are to be packed B to a bag, then how many bags are needed?
Arrays of Objects	*Equal groups language* Unknown Product There are A rows of apples with B apples in each row. How many apples are there?	Unknown Factor If C apples are arranged into A equal rows, how many apples will be in each row?	Unknown Factor If C apples are arranged into equal rows of B apples, how many rows will there be?
	Row and column language Unknown Product The apples in the grocery window are in A rows and B columns. How many apples are there?	Unknown Factor If C apples are arranged into an array with A rows, how many columns of apples are there?	Unknown Factor If C apples are arranged into an array with B columns, how many rows are there?
Compare	A > 1 Larger Unknown A blue hat costs \$$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs \$$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. How many times as much does the red hat cost as the blue hat?
	A < 1 Smaller Unknown A blue hat costs \$$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs \$$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. What fraction of the cost of the blue hat is the cost of the red hat?

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

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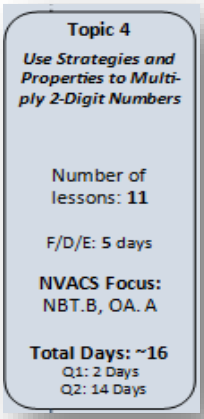
Van De Walle, J. A., Bay-Williams, J. M., Lovin, L. H., & Karp, K. S. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 6-8* (2nd ed.). New York, NY: Pearson.

▸ Grade 4 Topic 4: Use Strategies and Properties to Multiply by 2-Digit Numbers



Big Conceptual Idea: Number and Operations in Base Ten (pp. 12-15)

Prior to instruction, view the Topic 4 Professional Development Videos located in Pearson Realize online. Read the Teachers' Edition (TE): Cluster Overview/Math Background (pp. 43A-43F), the Topic Planner (pp. 167A-167D), all 11 lessons, and the Topic Assessments (pp. 243-248A).

Mathematical Background:	**Topic Essential Questions:**
Read Cluster Overview (TE, pp. 43A-43F)	How can you use a model to multiply? How can you use the Distributive Property to multiply? How can you use multiplication to solve problems? (TE, p. 167) *Reference TE (p. 167) and Answering the Topic Essential Questions (TE, pp. 243-244) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

4-1	4-2	4-3	4-4	4-5	4-6	4-7	4-8	4-9	4-10	4-11	Assessment

5 F/D/E days used strategically throughout the topic.

Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Instructional note:

This topic builds from Topic 3 by using place-value understanding and estimation to multiply multi-digit whole numbers. Focus instruction on Nevada Academic Content Standard (NVACS) 4.NBT.B.5 (2010). Emphasis for standard 4.NBT.B.5 is to "multiply two two-digit whole numbers using strategies based on place value and the properties of operations" (NVACS, 2010). The problem types or structures in this topic continue to focus on equal groups and multiplicative comparison when multiplying. As stated in the Topic 3 WCSD Curriculum Guide, Van de Walle, Karp, and Bay-Williams explain that equal group problems can be repeated addition or rate problems. Repeated addition problems such as, "If three children have four apples, how many apples are there?" and rate problems, 'If there are four apples per child, how many apples would three children have?'" (Van de Walle et al., 2010, pp. 154-155). Multiplicative comparison problems are when there are two different sets. Van de Walle, et al., state, "one set consists of multiple copies of the other. An example of a multiplicative comparison problem is, 'Jill picked 6 apples. Mark picked 4 times as many apples as Jill. How many apples did Mark pick?'" (2010, p. 155).

This topic focuses on the distributive, commutative and associative properties of multiplication, as well as the partial product algorithm using an area model or open array. Also included is the U.S. traditional algorithm. The U.S. traditional algorithm for multiplication is based on the Distributive Property. It is a very efficient procedure, yet also one of the most difficult algorithms for students to understand. Instruction that includes the conceptual knowledge behind procedures is crucial. Research shows that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Van de Walle, et al., state, "once having begun with traditional algorithms, it is extremely difficult to suggest to students that they learn other methods" (2010, p. 217). Be cautious introducing the U.S traditional algorithm without conceptual understanding.

Use of area models and the partial products algorithm rely on the distributive property and can help students develop the conceptual understanding necessary to become fluent with multi-digit multiplication. These algorithms allow students to see the partial products within the algorithms and how each is created, while avoiding the errors that often occur when regrouping and recording using the U.S. traditional algorithm. These algorithms also allow students to work horizontally or vertically, and to multiply factors and add partial products in varying order. You may consider using partial products algorithm instead of the U.S. traditional algorithm, especially until students are secure with the properties, models and partial products algorithm.

Area models and the partial products algorithm can be very efficient. Using these strategies, students create separate values for each partial product without additional time or writing when compared to the U. S. standard algorithm. The area model has many advantages over the U.S. traditional algorithm, especially as students begin using two-digit multipliers (Van de Walle, et al., 2014). Students should understand that multiple strategies could be used to solve these problems with some being more appropriate than others in certain instances. Effective instruction provides experiences that help students connect procedures with the underlying concepts (NCTM, 2014).

Estimation is important as students use estimation strategies to check for reasonableness to their answers. In this topic, students will also compare and round whole numbers. Rounding whole numbers is an **estimation** strategy. "The term estimation refers to a number that is a suitable approximation for an exact number given the particular context" (Van de Walle, et al., 2010, p. 241). Rounding is one strategy used to estimate. Number lines are useful tools to help students round numbers. Other estimation strategies include; compatible numbers, front-end methods, clustering and using tens and hundreds. Students should be able to use and recognize words and phrases for estimation like; about, approximately, close to; etc. Van de Walle, et al., (2010) say,

> Do not reward or emphasize the answer that is the closest. It is already very difficult for students to handle "approximate" answers; worrying about accuracy and pushing for the closest answers only exacerbates this problem. Instead, focus on whether the answers given are *reasonable* for the situation or problem at hand. (p. 242)

Students should be able to use rounding flexibly and understand it conceptually, so it can be a useful estimation strategy (Van de Walle, et al., 2010).

Compatible numbers are "two or three numbers that can be grouped to make benchmark values. If the numbers in the list can be adjusted slightly to produce these groups, this will make finding an estimate easier." (Van de Walle, et al., 2010, p. 247)

Karen Karp, Sarah Bush and Barbara Dougherty (2014), state the zero trick or multiply a number by ten, just add a zero to the end of the number expires because when students begin to multiple decimals in 5th grade this rule no longer applies. Be cautious when teaching the "zero trick" and consider avoiding it all together. Instead, consider connecting it to our place-value base ten system discussed in Topic 1.

Focus Math Practice 1: **Make sense and persevere**
Focus opportunities for students to develop *Mathematical Practice 1* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 4-11. Reference the Teacher's Edition (pp. F21-F21A) and the NVACS (2010, p. 6).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
compatible numbers	*Distributive Property* *Commutative Property of Multiplication* *Associative Property of Multiplication* *area model* *partial product algorithm*

Additional terminology that students may need support with: array or open array, estimate, rounding, factors, product, partial product, mental math, variable and bar diagram

*Consider using the additional terminology to label anchor charts used throughout this topic.

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students developing conceptual understanding and moving their thinking from less sophisticated understandings (place value blocks or arrays) towards a more sophisticated understanding (partial product algorithm; area model)?"
"Are students making connections between Topic 3 and Topic 4?"

Lesson	Evidence	Look for
4-5	***Intervention Activity*** (student work samples)	Focus CTC on the big idea: • student strategies and models. • student connection between Topic 3 and Topic 4.
4-9	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • student strategies and models connected to Topic 3. • difference in estimation strategies between rounding and compatible numbers; how the estimate leads to overestimation and underestimation.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 243-248	Use *Scoring Guide* TE pp. 243-248A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 4-1: Mental Math- Multiply by Multiples of 10		
4 NBT.B.5 **4.OA.A.3** MP.2 MP.7	**Access Prior Learning**: In Lesson 3-1, students multiplied whole numbers by multiples of 10, 100, and 1,000. In Topic 1, students also learned about the base-10 system and 10 times. **Developing the Big Idea:** In this lesson, students will use their previous knowledge to multiply two multiples of 10.	**Solve & Share:** Consider giving students the opportunity to use multiple tools or representations to complete the *Solve & Share*. **Visual Learning:** In the *Visual Learning Animation*, students will make connections in multiplication by using the associative property of multiplication. Consider making an anchor chart with the commutative and associative properties for multiplication. As in Topic 3, consider pausing animation to have an in-depth conversation regarding the property and why the "short-cut or zero trick" works. See the instructional note regarding the "zero trick". **Convince Me:** In the *Convince Me!*, students are asked to solve a missing factor multiplication. Consider facilitating a discussion around finding missing factors. Ask students, "What related fact could help us solve this problem?" **Another Example:** Consider using the *Another Example!* with all students as it asks students how the associative and commutative property of multiplication help with the patterns of zero. Facilitate a discussion around place-value understanding. **Independent Practice/Math Practices and Problem Solving:** Students do not need to do all the problems in their Student Edition (SE). Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. **Assess and Differentiate/Intervention Activity:** Be cautious when doing the *Intervention Activity* with students as it addresses the "zero trick". Instead, consider using place-value blocks to help students understand multiplying by two multiples of 10.
Lesson 4-2: Use Models to Multiply 2-Digit Numbers by Multiples of 10		
4.NBT.B.5 **MP.1** MP.2 MP.4 MP.5	**Access Prior Learning**: In lesson 3-5, students used an array to find partial products. This approach used place-value understanding and the distributive property. **Developing the Big Idea:** In this lesson, students will use place-value blocks, area models, arrays and partial products to multiply a 2-digit number by a multiple of 10.	**Look Back:** Consider facilitating a discussion around the *Look Back!* as it uses place-value understanding from Topic 1 and addresses our base-10 system of 10 times as many. **Visual Learning:** Students continue using an area model or open array to multiply 2-digit by 2-digits whole numbers. In the *Visual Learning Animation,* both an area model and open array are shown. Consider having students use both models and connect to Topic 3. **Independent Practice/Math Practices and Problem Solving:** Consider using items 13 and 14 as a review for Topic 2. You may also consider using these items as a grade. For item 17, consider having students estimate before solving and then facilitate a discussion around over/under estimate in regards to money. **Assess and Differentiate/Intervention Activity:** Consider facilitating a discussion around the *Intervention Activity* as it gives all students access to multiplying 2-digit whole numbers by 10 using place-value blocks.
Lesson 4-3: Estimate- Rounding		
4.NBT.B.5 **4.OA.A.3** MP.2 MP.3	**Access Prior Learning**: In previous topics, students used rounding as an estimation strategy to find sums, differences or products. **Developing the Big Idea:** In this lesson, students will use rounding to estimate the product of two 2-digit whole numbers.	**Solve & Share:** Consider having students share the different factors they used to solve for a product close to 1,400. Facilitate a conversation around which combination of numbers were an underestimate of 1,400, and which combinations were an overestimate to 1,400. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students. Consider using a number line as a tool to check for reasonableness.

Lesson 4-4: Estimate- Use Compatible Numbers

4.NBT.B.5

MP.2
MP.3

Access Prior Learning:
In third grade, students used compatible numbers to estimate sums and differences. In the previous lesson, students estimated products of two 2-digit numbers.

Beginning of the Big Idea:
In this lesson, students will use number sense and compatible numbers to estimate products.

Look Back:
Consider facilitating a discussion around the *Look Back!* as students are sharing their responses to the *Solve & Share*. You may also have students answer the *Look Back!* as they work on the *Solve & Share* and then facilitate a discussion. The *Look Back!* has students thinking about their number choices when estimating the product of 1,600.

Visual Learning:
The *Visual Learning Animation* discusses compatible numbers. Consider adding compatible numbers to an estimation anchor chart to show students there are multiple ways to estimate other than rounding. Compare compatible numbers to rounding.

Assess and Differentiate/Intervention Activity:
Consider using the *Intervention Activity* with all students as students make connections to compatible numbers and money. You may consider using this as Number String or Number Talk to see if students make connections to estimation and money on their own.

Lesson 4-5: Arrays and Partial Products

4.NBT.B.5
4.OA.A.3

MP.4
MP.7

Access Prior Learning:
In Lesson 3-5, students used an array to find partial products and added the partial products to find the final product.

Developing the Big Idea:
In this lesson, students will continue using arrays and the partial products algorithm to multiply two 2-digit whole numbers.

Look Back:
Consider facilitating a discussion around the *Look Back!*. The *Look Back!* brings in the concept of area through the context of the problem. Here is an opportunity to discuss area with students. Some may connect area from third grade to this problem. Students will have more opportunities to work with area in other topics.

Visual Learning:
Consider having students estimate before solving problems in the *Visual Learning Animation*.

Convince Me:
Consider facilitating a discussion with the whole class around the *Convince Me!*. The area model is given, students need to figure out the multiples and product from the area model. This problem may support students in future topics regarding area, as they will need to find the dimensions when they know the product.

Guided Practice:
Consider facilitating a discussion around item 3 as it asks students to check for reasonableness.

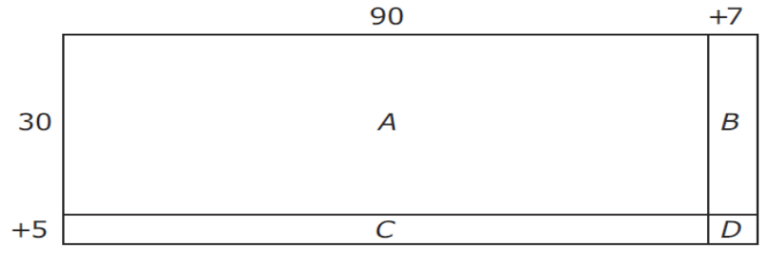
Assess and Differentiate/Intervention Activity:
Consider facilitating a discussion around the *Intervention Activity* with all students, because students are expanding from a one-digit by two-digit to a two-digit by two-digit. Students can make connections from Topic 3 to Topic 4.

Consider utilizing this question format during practice:

Example 2
Full Statement

Example Stem 2: In the area model shown, A = 2700 and D = 35. What are the values of B and C?

Enter your answer in the response box.



CTC: *Intervention Activity (student work samples)

Lesson 4-6: Multiply Using the Distributive Property

4.NBT.B.5

MP.4
MP.7

Access Prior Learning:
In third grade, students found the area of a rectangle. In previous lessons, students used an area model or open array to find the partial products.

Solve & Share:
The *Solve & Share* is another opportunity for students to see an area problem. Students should make connections to their prior learning of area and multiplication using an area model.

Look Back:
Consider using the *Look Back!* to facilitate a discussion around the partial products algorithm, from Topic 3. A student may have used the partial products algorithm to answer the *Solve & Share*, use this student's response to connect to the *Visual Learning Animation*.

-continues on next page-

	Developing the Big Idea: In this lesson, students will use the area model (open array) to extend their understanding of the Distributive Property and find the relationship between the two.	**Convince Me:** Consider using the *Convince Me!* as a whole class discussion, so students can see there are multiple ways to use the distributive property. List some of the ways students may have solved the problem from the *Visual Learning*. **Independent Practice/Math Practices and Problem Solving:** Consider using item 13 in a class discussion because it focuses on over/under estimation. **Assess and Differentiate/Intervention Activity:** Consider using place-value blocks or representations to support students who need scaffolding with distributive property. The following example is similar to *Guided Practice* item 3, consider utilizing this question format during practice: Example 1 Full Statement **Example Stem 1:** Enter the unknown number that makes the equation true. $26 \times 74 = (20 + 6) \times (\square + 4)$ Enter your answer in the response box.
Lesson 4-7: Use Partial Products to Multiply by 2-Digit Numbers		
4.NBT.B.5 **4.OA.A.3** MP.2 MP.3 MP.4 MP.7	**Access Prior Learning**: In the previous lessons, students learned and applied the partial products algorithm to multiply two 2-digit whole numbers. **Developing the Big Idea:** In this lesson, students will continue using the partial product algorithm to multiply multi-digit whole numbers.	**Solve & Share:** Consider removing the grid as to facilitate multiple student strategies. **Look Back:** Consider using the *Look Back!* to facilitate a discussion around estimation. Students should be estimating to check for reasonableness throughout topics and lessons. **Independent Practice/Math Practices and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider modifying a question to include the following phrase: Example 1 Full Statement **Example Stem:** Which strategy for multiplying 94 and 36 should result in the correct product? A. 94 × 36 = 24 + 540 + 120 + 270 B. 94 × 36 = 24 + 54 + 120 + 2700 C. 94 × 36 = 2700 + 540 + 120 + 240 D. 94 × 36 = 2700 + 540 + 120 + 24
Lesson 4-8: Multiply 2-Digit by 2-Digit Numbers		
4.NBT.B.5 **4.OA.A.3** MP.2 MP.7	**Access Prior Learning**: In Topic 3, students were exposed to the standard algorithm. **Developing the Big Idea:** In this lesson, students will use algorithms to multiply two 2-digit whole numbers.	**Solve & Share:** The *Solve & Share* is a multiplicative comparison problem. Note: this is the only multiplicative comparison problem in Topic 4. Consider this an opportunity to reintroduce the bar diagram if students did not use one. See Table 3, problem type is a compare, larger unknown. **Look Back:** Consider having students solve the *Look Back!* and facilitate a discussion regarding place-value understanding within the base-10 system when we multiply. **Visual Learning:** In the *Visual Learning Animation*, students are shown the U.S traditional algorithm. As stated in Topic 3, students do not need to be secure using the U.S. traditional algorithm. Consider showing students the strategy, but do not "expect" them to use it all the time. Give students multiple opportunities to continue using an area model, open array and the partial product algorithm to multiply. ***-continues on next page-***

		Convince Me: Again, consider using the *Convince Me!* to facilitate a discussion regarding our base-10 system and 10 times as many instead of focusing the discussion on the "zero trick". **Independent Practice/Math Practice and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate.
Lesson 4-9: Continue to Multiply By 2-Digit Numbers		
4.NBT.B.5 **4.OA.A.3** **MP.1** MP.3 MP.4 MP.7	**Access Prior Learning**: In Lesson 4-8, students compared the partial product algorithm to the U.S standard algorithm. **Developing the Big Idea:** In this lesson, students will continue using the algorithms to multiply whole numbers.	**Solve & Share:** Consider sharing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. **Look Back:** Consider having students answer the question in the *Look Back!*, and then facilitate a discussion around reasonableness by using estimation. Consider having students estimate throughout the lesson. **Visual Learning:** Consider having students continue estimating, and giving students the opportunity to use any strategy or model to solve the problems in the *Visual Learning Animation*. **Convince Me:** Consider using the *Convince Me!* to facilitate a discussion or used formatively to assess student understanding as students have had multiple opportunities to discuss place-value and the partial products algorithm. **Independent Practice/Math Practice and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. **Assess and Differentiate/Intervention Activity:** Consider giving students the opportunity to work on the partial products algorithm, area model or array, and not have students use the U.S. traditional algorithm. Consider modifying a question to include the following phrase: Example 1 Full Statement **Example Stem:** Which expression is equal to 36 × 94? A. (30 × 90) + (6 × 4) B. (30 + 6) × (90 + 4) C. (30 + 6) × 94 + (30 + 6) × 4 D. (30 × 90) + (30 × 6) + (90 × 6) + (90 × 4) ***CTC: *Solve & Share*** (student work samples)
Lesson 4-10: Continue to Multiply by 2-Digit Numbers		
4.NBT.B.5 **4.OA.A.3** **MP.1** MP.6 MP.7 MP.8	**Access Prior Learning**: In previous lessons and topics, students compared the partial product algorithm to the traditional algorithm. **Developing the Big Idea:** In the lesson, students will continue to use algorithms to multiply whole numbers.	**Solve & Share:** Consider sharing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. **Look Back:** Consider facilitating a discussion around the question as it asks students to think about an estimation strategy they may use or had used to solve the problem. Students may use compatible numbers and/or rounding to estimate. **Visual Learning:** Consider using the *Visual Learning Animation* area problem, but do not expect students to use the U.S. traditional algorithm. Consider accepting area models, arrays and the partial product algorithm. The *Visual Learning* problem is multi-step, as students will multiply and then subtract the two problems. Consider using this problem as a Gallery Walk, so students can critique the reasoning of others. ***-continues on next page-***

		Guided Practice: Item 1 is a missing factor multiplication problem within the U.S. traditional algorithm, consider giving on-level or above-level students this problem during independent practice, instead of doing whole class. **Independent Practice/Math Practice and Problem Solving:** Consider using item 23 to have students practice estimation strategies. Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider encouraging students to use multiple strategies to solve the problems. **Assess and Differentiate/Intervention Activity:** Consider giving students the opportunity to work on the partial products algorithm, area model or array, and not have students use the U.S. traditional algorithm.
Lesson 4-11: Math Practices and Problem Solving- Make Sense and Persevere		
4.NBT.B.5 **4.OA.A.3** **MP.1** MP.2 MP.4 MP.6 MP.7	**Access Prior Learning**: In previous lessons, students used multiple strategies to multiply two 2-digit whole numbers. **Developing the Big Idea:** In this lesson, students will continue to build fluency in multiplication using multiple strategies.	**Solve & Share:** Consider sharing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. **Look Back:** Consider having students work on the *Look Back!* as they work on the *Solve & Share*. **Visual Learning:** Students are asked to use a bar diagram and write an equation with a variable. Consider tying the *Convince Me!* to the *Visual Learning Animation* or discussion, as students are asked to solve the problem using different strategies. To make the connection, accept multiple strategies, but emphasize the distributive property. **Assess and Differentiate/Intervention Activity:** Consider using the problem given for the *Intervention Activity*, but do not expect students to use the U.S. traditional algorithm. Consider accepting strategies like the area model, open array or partial product algorithm. Some students may still need the support of place-value blocks, so help those students move from concrete to representational by drawing the models.

Table 3: Multiplication and division situations

	A × B = □	A × □ = C and C ÷ A = □	□ × B = C and C ÷ B = □
Equal Groups of Objects	Unknown Product There are A bags with B plums in each bag. How many plums are there in all?	Group Size Unknown If C plums are shared equally into A bags, then how many plums will be in each bag?	Number of Groups Unknown If C plums are to be packed B to a bag, then how many bags are needed?
Arrays of Objects	*Equal groups language* Unknown Product There are A rows of apples with B apples in each row. How many apples are there?	Unknown Factor If C apples are arranged into A equal rows, how many apples will be in each row?	Unknown Factor If C apples are arranged into equal rows of B apples, how many rows will there be?
	Row and column language Unknown Product The apples in the grocery window are in A rows and B columns. How many apples are there?	Unknown Factor If C apples are arranged into an array with A rows, how many columns of apples are there?	Unknown Factor If C apples are arranged into an array with B columns, how many rows are there?
Compare	$A > 1$ Larger Unknown A blue hat costs \$$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs \$$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. How many times as much does the red hat cost as the blue hat?
	$A < 1$ Smaller Unknown A blue hat costs \$$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs \$$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. What fraction of the cost of the blue hat is the cost of the red hat?

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

References

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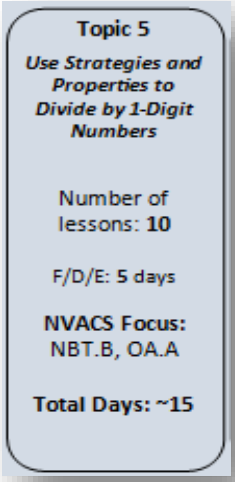
Grade 4 Topic 5: Use Strategies and Properties to Divide by 1-Digit Numbers



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Big Conceptual Idea: Number and Operations in Base Ten (pp. 12-15)

Prior to instruction, view the Topic 5 Professional Development Animation (located in Pearson Realize online). Read the Teacher Edition (TE), Cluster Overview/Math Background (pp. 43A-43F), the Topic Planner (pp. 249A-249D), all 10 lessons, and the Topic Assessments (pp. 323-324A).

Mathematical Background: Read Cluster Overview (TE, pp. 43A-43F)	**Topic Essential Questions:** How can mental math be used to divide? How can quotients be estimated? What tools, models or strategies can be used to divide? (This essential question was modified to focus on division conceptually.) *Reference the Teacher Edition (p. 249) and Answering the Topic Essential Questions (TE, p. 319-320) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

5-1	5-2	5-3	5-4	5-5	5-6	5-7	5-8	5-9	5-10	Assessment

5 F/D/E days used strategically throughout the topic.

Instructional note:

This topic focuses on dividing multi-digit whole numbers by using place-value understanding and estimation to divide. Focus instruction on Nevada Academic Content Standard (NVACS) 4.NBT.B.6. Emphasis for standard 4.NBT.B.6 is to "find whole-number quotients and remainders up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division" (NVACS, 2010). The U.S. traditional algorithm is not required until Grade 6, give students multiple opportunities to learn and practice division conceptually.

Van de Walle, Karp, & Bay-Williams (2010), state there are different division problem types, "Problems in which the size of the sets is unknown are called fair-sharing or partition problems. If the number of sets is unknown but the size of the equal sets is known, the problems are called measurement or sometimes, repeated subtraction problems" (p. 155). Although we note the different problem types of division throughout the lessons, this is background knowledge for teachers. At this time, students do not need to know the names of the different types of division, partitive and measurement.

In most situations in the real world, division does not end with an equal amount whole number. According to Van de Walle, et al., (2010), "in real contexts, remainders sometimes have three additional effects on the answer; the remainder is discarded (leaving a smaller whole number answer), the remainder can "force" the answer to the next highest whole number and the answer is rounded to the nearest whole number for an approximate result" (p. 155). Van de Walle, et al., (2010) goes on to say, "students should not just think of remainders as "R 3" or "left over". Remainders should be put in context and dealt with accordingly" (p. 156).

Estimation is important as students use estimation strategies to check for reasonableness to their answers. Students should be able to use and recognize words and phrases for estimation like; about, approximately, close to; etc. Rounding is one strategy used to estimate. Number lines are useful tools to help students round numbers. Other estimation strategies include; compatible numbers, front-end methods, clustering and using tens and hundreds. Compatible numbers are "two or three numbers that can be grouped to make benchmark values. If the numbers in the list can be adjusted slightly to produce these groups that will make finding an estimate easier" (Van de Walle, et al., 2010, p. 247).

Focus Math Practice 4: **Model with math**

Focus opportunities for students to develop *Mathematical Practice 4* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 5-10. Reference the Teacher's Edition (pp. F24-F24A) and the NVACS (2010, p. 7).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
remainder partial quotients	*equation* *divisor* *dividend* *quotient* *compatible numbers*

Additional terminology that students may need support with: estimate, round, area, divisible, division, hidden question

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students developing conceptual understanding and moving their thinking from less sophisticated understandings (place value blocks or arrays) towards a more sophisticated understanding (partial quotient algorithm; area model)?"
"Are students interpreting remainders based on the context of the problem?"

Lesson	Evidence	Look for
5-5	***Higher Order Thinking*** (student work samples) Item 13	Focus CTC on the big idea: • student strategies and models. • student interpretation of the remainder.
5-7	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • student connection between area model and partial quotients, as well as using properties of operations.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 319-324	Use *Scoring Guide* TE pp. 319-324A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 5-1: Mental Math- Find Quotients		
4 NBT.B.6 MP.2 MP.3 **MP.4** MP.7	**Access Prior Learning**: In third grade, students learned basic division facts and division strategies. In previous topics, students learned to use patterns within our Base Ten System to multiply numbers. **Beginning of the Big Idea:** In this lesson, students will learn to divide multiples of 10, 100 and 1,000 by one-digit numbers, using the inverse work they did in Topic 3.	**Solve & Share:** The *Solve & Share* is a partitive division problem. Students may be seen dealing out the 270 "hockey cards" one by one into the 9 boxes. Consider giving students the opportunity to use multiple tools or representations to complete the *Solve & Share*. Consider having students share and compare strategies who may have dealt using concrete tools, used a representation or solved abstractly by using a derived or known facts to solve the problem. **Look Back:** Consider having a discussion around the inverse operation of division, which is multiplication, when students write an equation 9 X n = 270. Have students understand they were finding 9 groups of *n* = 270. **Visual Learning:** The *Visual Learning* problem is a partitive division problem. Consider giving students the opportunity to use multiple tools or representations during the *Visual Learning Animation*. **Convince Me:** The *Convince Me!* presents three missing dividend problems. This may be the first time students have had experience with this type of division problem, this idea connects to place-value patterns and basic fact multiplication. Consider asking, "What basic facts can you use to solve the problem?" **Independent Practice/Math Practices and Problem Solving:** Students do not need to do all the problems in their Student Edition (SE). Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. ***-continues on next page-***

		Assess and Differentiate/Intervention Activity: Be cautious when doing the *Intervention Activity* with students as it addresses the "zero trick". Instead, consider using place-value blocks to help students understand the patterns in place-value.
Lesson 5-2: Mental Math- Estimate Quotients		
4.NBT.B.6 MP.2 MP.3 **MP.4**	**Access Prior Learning**: In the previous topic, students used compatible numbers to estimate the product. **Developing the Big Idea:** In this lesson, students will think about multiplication and use compatible numbers to find the quotient with a 1-digit divisor.	**Solve & Share:** The *Solve & Share* is an estimation partitive division problem. Students may deal out the 248 tickets to 3 people one by one. Structure the discussion around students who may have used strategies of estimation. **Look Back:** Consider facilitating a discussion around the *Look Back!* as estimation is the focus and how it relates to division. **Visual Learning:** The *Visual Learning* is an estimation partitive division problem. **Convince Me:** Consider using the *Convince Me!* to facilitate a discussion regarding estimation strategies that may work best for the given situation. In the *Convince Me!* compatible numbers and rounding are compared. Estimation in division often involves compatible numbers as opposed to rounding. Ask students, "What is an easy number to divide by?" Consider reading the Prevent Misconception at the bottom of TE p. 260, as it explains there may be more than one way to use compatible numbers to find a quotient. **Guided Practice:** Consider having students complete items 1 & 2 in the *Guided Practice* and then facilitate a discussion around the item. Item 1 has students begin to think about the situation and remainders (leftovers). Consider also completing items 3, 4, 8 & 9 as the problems give students an opportunity to use compatible numbers and to think about the inverse operation, multiplication. **Independent Practice/Math Practices and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate.
Lesson 5-3: Mental Math- Estimate Quotients for Greater Dividends		
4.NBT.B.6 MP.2 MP.3	**Access Prior Learning**: In the previous lesson, students estimated quotients by using various estimation strategies. **Developing the Big Idea:** In this lesson, students will continue estimating quotients.	**Solve & Share:** The *Solve & Share* is an estimation, measurement division problem. In measurement division problems, students chunk out the number in each group to figure out how many groups are needed or made. Look for students who may have used estimation, and did not find the exact amount of groups. **Visual Learning:** The *Visual Learning* is an estimation partitive division problem. Students use compatible numbers to estimate, and derive multiplication facts to solve the problem. **Another Example:** Consider using the *Another Example!* with the whole class as rounding in division is the focus. Compare what students learned in the *Visual Learning Animation* to the *Another Example!*. Read the note on bottom of the Teacher's Edition about the *Another Example!* before the lesson (pp. 267-268). **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students, especially if students are having difficulty with the division in the lesson. The *Intervention Activity* uses estimation and multiplication to divide quotients with greater dividends.
Lesson 5-4: Interpret Remainders		
4.NBT.B.6 MP.1 MP.2 MP.3 MP.4	**Access Prior Learning**: In third grade, students learned how to divide with basic facts. In previous lessons, students used basic facts to divide multiples of 10, 100 and 1,000.	**Solve & Share:** The *Solve & Share* is a measurement division problem. Students will chunk the 4 apples and group them to fill a basket. Child-watch for students as they interpret the 2 remaining apples. Students should conclude they cannot make another full basket. **Visual Learning:** The *Visual Learning* is a measurement division problem. Students interpret the remainder or leftover whole number". Consider discussing the context of the problem, how this helps to determine what to do with the remaining amount or leftover. Consider creating an anchor chart for the three different interpretations of remainders. ***-continues on next page-***

	Beginning of the Big Idea: In this lesson, students will use models or drawings to find quotients involving remainders.	**Convince Me:** Consider using the *Convince Me!* as a formative assessment to see if students are using derived facts to determine where the mistake was made. **Another Example:** The *Another Example!* uses counters to make a model representation. Consider having students use the concrete tools or draw the representation to support their understanding of division. **Guided Practice:** Consider facilitating a whole class discussion around items 1 and 2. By doing these items, students begin to think about remainders and what to do with those remainders: keep them, drop 1 or add one. Item 2 also gives students more opportunity to work with measurement division problems. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students as it is a partitive division problem and supports students' conceptual knowledge of division. Consider supporting students who need more scaffolding regarding a partitive or dealing division problem with remainders.
Lesson 5-5: Division as Sharing		
4.NBT.B.6 **4.OA.A.3** MP.2 MP.3 **MP.4** MP.5	**Access Prior Learning**: In third grade, students learned the concepts of division as sharing and as repeated subtraction. In the previous lesson, students used models and representations to find quotients with remainders. **Developing the Big Idea:** In this lesson, students use representations to find quotients, with or without remainders.	**Solve & Share:** The *Solve & Share* is a partitive division problem as students deal patches into rows. Consider having students who used concrete tools (like counters) and representations (grouping) to solve the problem share their strategies or models. Structure the discussion around an efficient strategy, like mental math rather than long division algorithm. Consider using the *Solve & Share* as a Number Talk. **Look Back:** Consider facilitating a discussion around the *Look Back!* as students discuss dividing objects into equal groups, but have them think about what happens when objects are not divided equally. **Visual Learning:** The *Visual Learning* is a partitive division problem. Consider having students use place-value blocks or other representations to show dealing. **Convince Me:** Consider facilitating a discussion with the whole class around the *Convince Me!* or as a formative assessment, because it is a real-world problem with regrouping money. **Another Example:** The *Another Example!* shows regrouping to divide 55 into 4 groups. Consider having a whole class discussion around this problem while students use tools or representations to find the quotient. **Independent Practice/Math Practices and Problem Solving:** Consider having students estimate for items 5-8 before solving for the actual quotient. Consider facilitating a discussion around item 9, as students will use a bar diagram and write an equation to solve the problem. This gives students an opportunity to model with math using other representations. ***CTC**: ***Higher Order Thinking item 13*** (student work samples)
Lesson 5-6: Use Partial Quotients to Divide		
4.NBT.B.6 **4.OA.A.3** MP.1 MP.2 **MP.4** MP.7	**Access Prior Learning**: In third grade, students learned division as sharing and as repeated subtraction. **Beginning of the Big Idea:** In this lesson, students will use repeated subtraction and partial quotients algorithm to divide.	**Solve & Share:** The *Solve & Share* is a measurement division problem where students chunk 6 out of 72 to find how many times Sally's bird feeder can be filled. Consider having students who used place-value understanding or repeated subtraction to find the quotient share their strategies or models. **Visual Learning:** The *Visual Learning* is another measurement division problem, and encourages students to think about partial quotients. Consider having students think about ways they may chunk 63 ÷ 3 before sharing the animation. Consider creating an anchor chart with the partial quotients algorithm and area model. **Convince Me:** Consider using the *Convince Me!* as a formative assessment to child-watch for students who understand the multiplication and division relationship. ***-continues on next page-***

		Another Example: Consider facilitating a discussion around the *Another Example!* as the focus is using the distributive property to divide. Students will draw an array to model the distributive property. **Guided Practice:** Consider having students complete items 1-3 in the *Guided Practice* as students use models and the distributive property to solve for the quotients. Consider modifying a question to include the following phrase: **Example Stem:** Enter the unknown number to make the equation true. $98 \div 5 = (\square \div 5) + (8 \div 5)$
Lesson 5-7: Use Partial Quotients to Divide- Greater Dividends		
4.NBT.B.6 **4.OA.A.3** MP.2 MP.3 MP.7	**Access Prior Learning**: In the previous lesson, students used repeated subtraction and the partial quotients algorithm to divide. **Developing the Big Idea:** In this lesson, students will build on the work from the previous lesson as well as estimate to solve division problems.	**(Possible 2-day Lesson)** **Note:** Consider expanding this lesson over 2 days to elicit more time for students to work with the area model and partial quotients algorithm. Remember by taking two days for this lesson, it will be one of the F/D/E days on the WCSD Pacing Framework. **Day 1:** **Solve & Share:** The *Solve & Share* is an area problem, where students have to solve for one of the dimensions. Here is another opportunity to facilitate a discussion around area after students have solved the problem to prepare them for later topics. **Look Back:** Consider using the *Look Back!* to facilitate a discussion, or have students work together to solve for the answer to the problem. **Visual Learning:** Consider facilitating a discussion around the *Visual Learning Animation*, as students will interpret the quotient and remainder. Consider having students compare both the area model and partial quotients algorithm to make connections. **Day 2:** **Another Example:** Consider facilitating a discussion around the *Another Example!* as it asks students to use multiplication, estimation, place-value and an open array to solve for the quotient. Child-watch for ways students use various multiplication, estimation and place-value to solve the problem. Have students share the different ways. **Independent Practice/Math Practices and Problem Solving:** Consider having a discussion around items 14 and 15 as students use information from a table to solve problems; as well as use estimation. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students, as they are asked to use partial quotients and an open area model to solve a division problem. Have students compare the model and algorithm. Look for students who notice the place-value connection. Consider asking students if there are other ways to find the quotient using an open area model or partial quotient algorithm. ***CTC: *Intervention Activity*** (student work samples)
Lesson 5-8: Divide with 1-Digit Numbers		
4.NBT.B.6 **4.OA.A.3** MP.2 MP.3 **MP.4** MP.6	**Access Prior Learning**: In the previous lessons, students learned how to divide by using the partial quotients algorithm. **Beginning of the Big Idea:** In this lesson, students will learn to divide by using the partial quotient algorithm.	**Note: Students will not need to know the U.S. traditional division algorithm for the Topic Performance Assessment, as this algorithm is not required until Grade 6. Consider spending time developing conceptual understanding by using the partial quotient algorithm or models.** **Solve & Share:** The *Solve & Share* is a partitive division problem. Give students opportunity to use concrete tools or representational models to solve the problem. Consider having students share who may have used concrete tools, models, or the partial quotients algorithm in this order. Have students compare the tools, models or strategies. ***-continues on next page-***

		Look Back: Consider having students solve the *Look Back!* and facilitate a discussion regarding how to construct an argument. **Visual Learning:** In the *Visual Learning Animation*, students will be exposed to the U.S. traditional division algorithm. Rather than showing the animation, have students use the partial quotients algorithm to solve the problem. Students may need more time to develop the conceptual understanding of division before moving into a procedural understanding. Students will have more opportunity with the U.S. traditional division algorithm in future grades. **Another Example:** Consider giving students more opportunities to work with the partial quotients algorithm using an open array instead of the U.S. traditional division algorithm. **Independent Practice/Math Practice and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider giving students opportunities to estimate and use place-value strategies to solve problems in the *Independent Practice and Math Practice and Problem Solving*.
Lesson 5-9: Continue to Divide with 1-Digit Numbers		
4.NBT.B.6 MP.2 MP.3 MP.7	**Access Prior Learning**: In the previous lesson, students continued developing the conceptual understanding of division using the partial quotient algorithm. **Developing the Big Idea:** In this lesson, students will continue using the partial quotient algorithm to solve division problems.	**Note: Students will not need to know the U.S. traditional division algorithm for the Topic Performance Assessment, and this algorithm is not required until Grade 6. Consider spending time developing conceptual understanding by using the partial quotient algorithm or models.** **Solve & Share:** The *Solve & Share* is a partitive division problem. Consider having students share and compare multiple strategies used to find the quotient**.** Have students share the concrete tools, representation (area model) and the partial quotient algorithm (abstract) in that order. **Look Back:** Consider having students answer the question in the *Look Back!*, and then facilitate a discussion in regards to patterns in multiplication and division. **Visual Learning:** The *Visual Learning* is a measurement division problem. Estimation is an important concept for students, so consider having students continue estimating. Give students an opportunity to use any strategy or model to solve the problems in the *Visual Learning Animation,* as the animation continues exposing students to the U.S. traditional division algorithm. **Convince Me:** Consider using the *Convince Me!* to facilitate a discussion around estimation strategies. Have students think about place value as they solve the problem. **Independent Practice/Math Practice and Problem Solving:** Students do not need to do all the problems in their Student Edition. Continue on to other items as appropriate. Consider giving students opportunity to use any strategy or model they want to solve the division problems. Have students complete items 20 and 22 as they reinforce estimation. **Assess and Differentiate/Intervention Activity:** Consider having all students do the *Intervention Activity*, as students continue work with compatible numbers to estimate each quotient. Consider modifying a question to include the following example: **Example Stem:** Select the equation that has the same unknown number as 90 ÷ 5 = □. A. 5 × 90 = □ B. 90 × □ = 5 C. 5 × □ = 90 D. □ × 90 = 5

Lesson 5-10: Math Practices and Problem Solving- Model with Math		
4.NBT.B.6 **4.OA.A.3** **MP.4** MP.1 MP.2 MP.6 MP.7 MP.8	**Access Prior Learning**: In Topic 3, students learned thinking habits that good problem solvers use to model with math. **Developing the Big Idea:** In the lesson, students will continue to use thinking habits when modeling with math for division.	**Solve & Share:** Consider sharing multiple strategies students used to answer the *Solve & Share*. Consider sharing the concrete strategies, then representational and algorithms (abstract) in that order. **Look Back:** Consider facilitating a discussion around hidden questions. To find some answers, the hidden questions need to be exposed in order to solve the problem. Hidden questions are something not explicitly asked, but which must be done before the explicit question can be answered. In Topic 6, students will have more opportunities with hidden questions. **Independent Practice/Math Practice and Problem Solving:** Consider having students complete items 3 and 4, as 4 asks to check for reasonableness. **Assess and Differentiate/Intervention Activity:** Consider doing the *Intervention Activity* with all students as it reinforces the bar diagram model. Students also find the hidden question to solve the problem. They will be doing more of this in Topic 6.

Table 3: Multiplication and division situations

	A × B = □	A × □ = C and C ÷ A = □	□ × B = C and C ÷ B = □
Equal Groups of Objects	Unknown Product There are A bags with B plums in each bag. How many plums are there in all?	Group Size Unknown If C plums are shared equally into A bags, then how many plums will be in each bag?	Number of Groups Unknown If C plums are to be packed B to a bag, then how many bags are needed?
Arrays of Objects	*Equal groups language* Unknown Product There are A rows of apples with B apples in each row. How many apples are there?	Unknown Factor If C apples are arranged into A equal rows, how many apples will be in each row?	Unknown Factor If C apples are arranged into equal rows of B apples, how many rows will there be?
	Row and column language Unknown Product The apples in the grocery window are in A rows and B columns. How many apples are there?	Unknown Factor If C apples are arranged into an array with A rows, how many columns of apples are there?	Unknown Factor If C apples are arranged into an array with B columns, how many rows are there?
Compare	$A > 1$ Larger Unknown A blue hat costs \$$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs \$$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. How many times as much does the red hat cost as the blue hat?
	$A < 1$ Smaller Unknown A blue hat costs \$$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs \$$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. What fraction of the cost of the blue hat is the cost of the red hat?

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

References

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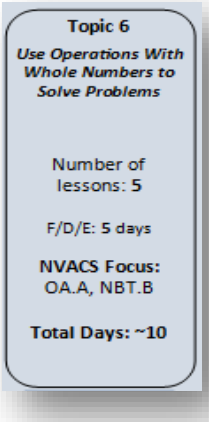
Grade 4 Topic 6: Use Operations with Whole Numbers to Solve Problems



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: Operations and Algebraic Thinking (pp. 29-31)
Prior to instruction, view the Topic 6 Professional Development Animation located in Pearson Realize online. Read the Teacher Edition (TE): Cluster Overview/Math Background (pp. 325A-325F), the Topic Planner (pp. 325I-325J), all 5 lessons, and the Topic Assessments (pp. 363-364A).

Mathematical Background:	**Topic Essential Questions:**
Read Cluster Overview (TE, pp. 325A-325E)	How is comparing multiplication different from comparing addition? How can you use equations to solve multi-step problems? *Reference TE (p. 325) and Answering the Topic Essential Questions (TE, pp. 361-362) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

6-1	6-2	6-3	6-4	6-5	Assessment

5 F/D/E days used strategically throughout the topic

Instructional note:
This topic focuses on comparison type problems, especially multiplicative comparison and solving multi-step problems. Focus instruction on Nevada Academic Content Standards (NVACS) 4.OA.A.1, 4.OA.A.2 and 4.OA.A.3. Emphasis for standard 4.OA.A is the use of four operations with whole numbers to solve problems. "Comparison situations involve two distinct sets or quantities and the difference between them. In multiplicative comparison problems, there are really two different sets, as there were with comparison situations for addition and subtraction. One set consists of multiple copies of the other" (Van de Walle, Karp, Bay-William, 2010, p. 155). Multiplicative comparison problems do not contain an action within the problem itself making this difficult for students working in the concrete or direct model phases.

In solving multi-step problems, students sometimes find solving multi-step problems difficult. Van de Walle, et al., suggest when giving students multi-step problems consider using the following ideas to show students how multi-step problems "are chained together;

- first give students a one-step problem and have them solve it
- make hidden questions and have students identify the questions
- pose standard two-step problems and have students identify the hidden question

(2010, p. 163-164).

Focus Math Practice 1: **Make sense and persevere**
Focus opportunities for students to develop *Mathematical Practice 1* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 6-5. Reference the Teacher's Edition (pp. F21-F21A) and the NVACS (2010, p. 6).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
	variable *equation*

Additional terminology that students may need support with: comparison, hidden questions, square units

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students persevering and making sense of multi-step problems?"

Lesson	Evidence	Look for
6-3	***Quick Check*** (digital platform) Item 5	Focus CTC on the big idea: • student reasoning around multi-step problem solving. Printable version available under "Teacher Resources".
6-5	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • student reasoning around multi-step problem-solving. • student sense making and persevering.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 361-364	Use *Scoring Guide* TE pp. 361-364A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 6-1: Solve Comparison Situations		
4.OA.A.1 **4.OA.A.2** **4.NBT.B.5** **MP.1** MP.2 MP.3 MP.4 MP.5 MP.6	**Access Prior Learning**: In previous grades and topics, students developed computational skills using various strategies, models or algorithms to solve problems. **Developing the Big Idea:** In this lesson, students will use various skills to solve problems involving comparisons.	**Solve & Share:** Consider removing the bar diagram from the *Solve & Share* to elicit additional student strategies or models. Consider having students who used the bar diagram share their strategies. If students did not use a bar diagram, consider showing the bar diagram model that "a student from last year used". **Look Back:** Have students think about the equation that can be written from the problem. Have a discussion based on the "4 times as long" being a multiplicative comparison problem. Reference back to the bar diagram and ask students why the problem may be a multiplicative comparison problem. **Visual Learning:** In the *Visual Learning Animation*, the problem is shown as a multiplicative comparison and as an addition comparison. Consider facilitating a discussion by comparing the two types of comparison problem types (additive and multiplicative). **Convince Me:** Consider facilitating a discussion around the *Convince Me!,* so students can continue comparing a multiplicative comparison with an addition comparison problem. Connect the *Convince Me!* to the *Solve & Share* and *Visual Learning Animation*. **Guided Practice:** **Note:** The Error Intervention focuses in on key word strategies to tell the difference between a multiplicative comparison and addition comparison (TE, pp. 329-330). Consider giving students opportunity to find and explain the difference between these comparison types on their own rather than teaching key word strategies.
Lesson 6-2: Continue to Solve Comparison Situations		
4.OA.A.2 **4.OA.A.1** **4.NBT.B.5** **4.NBT.B.6** **MP.1** MP.2 MP.3 MP.4	**Access Prior Learning**: In the previous lesson, students learned to distinguish between comparison situations involving multiplication and addition. **Developing the Big Idea:** In this lesson, students will continue solving multiplicative comparison problems, including some that require division to solve.	**Solve & Share:** Consider removing the sentence frames and "Complete the sentences and equation to show a way to compare the height of the tree". Doing so, will increase the cognitive demand and elicit more strategies and different comparisons to solve the problem. For example, some students may use an additive comparison or multiplicative comparison. Whereas others may use division to compare the heights. **Convince Me:** Consider facilitating a discussion around the *Convince Me!*, as students are asked to determine when division is used to comparison situations. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students, as students are asked to use a bar diagram to solve a multiplicative comparison problem.

Lesson 6-3: Solve Multi-step Problems		
4.OA.A.3 **4.OA.A.1** **4.NBT.B.5** **4.NBT.B.6** **MP.1** MP.4 MP.7 MP.8	**Access Prior Learning**: In previous topics, students developed computational skills by using models and equations to solve problems. **Developing the Big Idea:** In this lesson, students will use computational skills, modeling and problem solving to solve multi-step problems using all four operations.	**Solve & Share:** Consider giving students the opportunity to use concrete tools, various representations or models and other strategies to solve the multi-step problem. **Look Back:** Consider facilitating a discussion around the *Look Back!* as the question ties to estimation and mental math to check for reasonableness. Students should have multiple opportunities to estimate. **Visual Learning:** The *Visual Learning* addresses hidden questions in problems. Hidden questions can be difficult for some students to find in a multi-step problem, though they had some exposure to it in topic 5 or previous grades. Give students opportunities to use tools and/or models to support their understanding of multi-step problems. Some students might realize they only need to do 2 X 18 to find the difference. **Independent Practice/Math Practices and Problem Solving:** Students do NOT have to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. ***CTC: *Quick Check*** (digital platform)
Lesson 6-4: Solve More Multi-step Problems		
4.OA.A.3 **4.NBT.B.5** **4.NBT.B.6** **MP.1** MP.3 MP.4	**Access Prior Learning**: In previous topics, students developed computational skills by using models and equations to solve problems. In the previous lesson, students used computational skills, modeling and problem solving to solve multi-step problems using all four operations. **Developing the Big Idea:** In this lesson, students will extend their work to solving problems with more than two steps.	**Solve & Share:** Consider encouraging students to use concrete tools, various representations, models or other strategies to solve the multi-step problem. Look for students who used the partial product algorithm to solve the problem. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students, as students use multiple bar diagrams to solve multi-step problems.
Lesson 6-5: Math Practices and Problem Solving- Make Sense and Persevere		
4.OA.A.2 **4.OA.A.3** **4.NBT.B.5** **4.NBT.B.6** **MP.1** MP.2 MP.4 MP.6	**Access Prior Learning**: In previous lessons and topics, students have had to persevere to solve multi-step problems using the four operations. **Developing the Big Idea:** In this lesson, students will persevere in finding hidden questions and solving multi-step problems using the four operations.	**Solve & Share:** The *Solve & Share* is an area problem. Consider encouraging students to use tools or representations, like a grid to support them in solving the problem. ***CTC: *Solve & Share*** **(**student work samples)

Table 2: Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$	A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first A bunnies? $A + \square = C$	Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before? $\square + B = C$
Take From	C apples were on the table. I ate B apples. How many apples are on the table now? $C - B = \square$	C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat? $C - \square = A$	Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before? $\square - B = A$
	Total Unknown	**Both Addends Unknown[1]**	**Addend Unknown[2]**
Put Together /Take Apart	A red apples and B green apples are on the table. How many apples are on the table? $A + B = \square$	Grandma has C flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$	C apples are on the table. A are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	*"How many more?" version.* Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy? *"How many fewer?" version.* Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$	*"More" version suggests operation.* Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have? *"Fewer" version suggests wrong operation.* Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have? $A + B = \square$	*"Fewer" version suggests operation.* Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have? *"More" version suggests wrong operation.* Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$

Table 3: Multiplication and division situations

	$A \times B = \square$	$A \times \square = C$ and $C \div A = \square$	$\square \times B = C$ and $C \div B = \square$
Equal Groups of Objects	Unknown Product There are A bags with B plums in each bag. How many plums are there in all?	Group Size Unknown If C plums are shared equally into A bags, then how many plums will be in each bag?	Number of Groups Unknown If C plums are to be packed B to a bag, then how many bags are needed?
Arrays of Objects	*Equal groups language* Unknown Product There are A rows of apples with B apples in each row. How many apples are there?	Unknown Factor If C apples are arranged into A equal rows, how many apples will be in each row?	Unknown Factor If C apples are arranged into equal rows of B apples, how many rows will there be?
	Row and column language Unknown Product The apples in the grocery window are in A rows and B columns. How many apples are there?	Unknown Factor If C apples are arranged into an array with A rows, how many columns of apples are there?	Unknown Factor If C apples are arranged into an array with B columns, how many rows are there?
Compare	$A > 1$ Larger Unknown A blue hat costs \$$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?	Smaller Unknown A red hat costs \$$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. How many times as much does the red hat cost as the blue hat?
	$A < 1$ Smaller Unknown A blue hat costs \$$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?	Larger Unknown A red hat costs \$$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?	Multiplier Unknown A red hat costs \$$C$ and a blue hat costs \$$B$. What fraction of the cost of the blue hat is the cost of the red hat?

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B, so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Van De Walle, J., Karp, K., & Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally* (7th ed.). Boston, MA: Pearson.

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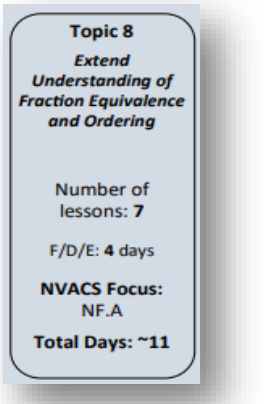
▸ Grade 4 Topic 8: Extend Understanding of Fraction Equivalence and Ordering



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: Number and Operations- Fractions (pp. 121-125)
Prior to instruction, view the Topic 8 Professional Development Animation located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 407A-407F), the Topic Planner (pp. 407I-407K), all 7 lessons, and the Topic Assessments (pp. 459-460A).

Mathematical Background:	**Topic Essential Questions:**
Read Cluster Overview- (TE, pp. 407A-407F)	What are some ways to name the same part of a whole? How can you compare fractions with unlike denominators? *Reference TE p. 407 and Answering the Topic Essential Questions (TE, pp. 457-458) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

8-1	8-2	8-3	8-4	8-5	8-6	8-7	Assessment

4 F/D/E days used strategically throughout the topic.

Instructional note:
This topic focuses on finding equivalent fractions and comparing fractions. Focus instruction on Nevada Academic Content Standards (NVACS) 4.NF.A.1. and 4.NF.A.2. Emphasis for standard 4.NF.A, is to "extend understanding of fraction equivalence and ordering" (2010). Students will find equivalent fractions "by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size" (NVACS, 2010, 4.NF.A). In Grade 3, students worked with denominators of 2, 3, 4, 6 and 8. In Grade 4, students will add in denominators of 5, 10, 12 and 100.

"A key idea about fractions that students must come to understand is that a fraction does not say anything about the size of the whole or the size of the parts. A fraction tells us only the *relationship between* the part and the whole" (Van de Walle, Karp, Bay-Williams, 2010, p. 288). When students compare fractions, they need to consider that both fractions are part of the same whole. A common misconception students have when comparing fractions is that "a/b > c/d if a > c and b > d; in other words, they think that a fraction with both a greater numerator and a greater denominator than another fraction has to be greater than that second fraction" (Small, 2104, p. 44). Using various tools, models, benchmark fractions and strategies to compare fractions will help students in developing their fractional knowledge.

Models are important when students are beginning to understand fractional concepts. The different models give students various opportunities to learn fractions. These different models include; region or area models, length models or number line models and set models. For example, "an area model helps students visualize parts of the whole. A linear model shows that there is always another fraction to be found between any two fractions-an important concept that is underemphasized in the teaching of fractions" (Van de Walle, et al., 2010, p. 288). Van de Walle, et al., continue to emphasize the use of models, "It is important to remember that students must be able to explore fractions across models. If they never see fractions represented by length, they will struggle to solve any problem or context that is linear. As a teacher you will not know if they really understand the meaning of fractions unless they can model a fraction using different context or models" (2010, pp. 290-291).

"In a problem-based classroom, students can develop an understanding of equivalent fractions and also develop from that understanding a conceptually based algorithm" (Van de Walle, et al., 2010, p. 302). Students need to understand equivalence, because "it is a critical but often poorly misunderstood concept" (Van de Walle, et al., 2010, p. 301). Equivalent fractions name two different names for the same point in a number line and same sized parts of the whole.

Students need a conceptual understanding of equivalent fractions before a procedural understanding. A conceptual understanding of equivalent fractions is, "two fractions are equivalent if they are representations for the same amount or quantity-if they are the same number" (Van de Walle, et al., 2010, p. 302). A procedural understanding of equivalent fractions is "to get an equivalent fraction, multiply or divide to top and bottom numbers by the same nonzero number" (Van de Walle, et al., 2010, p. 302). Giving students multiple opportunities with models or representations will help students build a conceptual understanding for finding equivalent fractions. While some of these lessons teach procedurally to find equivalent fractions, continue teaching the conceptual understanding of why fractions are equivalent.

Focus Math Practice 3: **Construct arguments**
Focus on opportunities for students to develop *Mathematical Practice 3* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 8-7. Reference the Teacher's Edition (TE, pp. F23-F23A) and the NVACS (2010, p. 6).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
common factor benchmark fraction	*equivalent fraction* *fraction* *numerator* *denominator* *Identity Property of Multiplication*

Additional terminology that students may need support with: unit fraction, equality, inequality

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students able to name the same part of a whole?"
"Are students able to compare fractions using various strategies?"

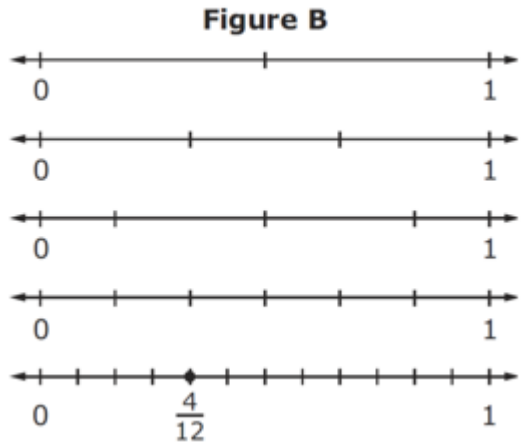
Lesson	Evidence	Look for
8-1	***Convince Me!*** (student work samples)	Focus CTC on the big idea: • students recognize equivalent fractions based on the same size whole.
8-6	***Quick Check*** (digital platform) Items 3 and 5	Focus CTC on the big idea: • students' understanding around the comparison of fractions. Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 457-460	Use *Scoring Guide* TE pp. 457-460A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 8-1: Equivalent Fractions- Area Model		
4.NF.A.1 MP.1 MP.2 MP.5	**Access Prior Learning**: In third grade, students used an area model to find simple equivalent fractions and explain why they are equivalent. **Developing the Big Idea:** In this lesson, students will find equivalent fractions using an area model and determine why they are equivalent.	**Note:** For fractions to be equivalent, the equivalent fractions must name the same part of the same whole. **Convince Me:** Consider changing the *Solve & Share* to the *Convince Me!* problem, "Mia ate ¼ of a pizza. Matt ate 2/8 of another pizza. Did Mia and Matt eat the same amount of pizza? Explain." This will elicit more strategies from students. Consider having concrete tools, like fraction strips available for students to solve the problem. Read *Convince Me!* note in the Teacher's Edition for an explanation about the *Convince Me!* (p. 412) problem. Child-watch for students who may have misconceptions when it comes to equivalent fractions. **Solve & Share:** After students answer the *Convince Me!* and strategies are shared, consider having students work together on the *Solve & Share* to find equivalent fractions. Consider including the *Look Back!* question, "How do you know your fraction is equivalent to ¼?" as the discussion around the *Solve & Share* is taking place. **Visual Learning:** Students are asked to use an area model to explain equivalency between 5/6 & 10/12. Consider having students use fraction strips along with the area model to support the conceptual understanding of equivalent fractions. ***-continues on next page-***

		Math Practices & Problem Solving: Consider modifying item 21 to include the following example format during practice: Example 1 Full Statement **Example Stem:** Figure A has $\frac{2}{3}$ of its whole shaded gray.  **Figure A** Decide if each fraction is equal to $\frac{2}{3}$. Select Yes or No for each fraction. $\frac{4}{6}$ $\frac{1}{2}$ $\frac{8}{12}$ **Assess and Differentiate/Intervention Activity:** Consider facilitating a discussion around the *Intervention Activity* with all students. The activity has students compare fraction strips. You may want to consider having students make their own construction paper fraction strips kit. ***CTC: *Convince Me!*** (student work samples)
Lesson 8-2: Equivalent Fractions- Number Lines		
4.NF.A.1 MP.1 **MP.3** MP.4 MP.5 MP.7	**Access Prior Learning**: In third grade, students used number lines to find simple equivalent fractions and to explain why they are equivalent. **Developing the Big Idea:** In this lesson, students continue to find equivalent fractions by using the number line and area model. **Look Ahead:** Emphasize number lines, as they will be an important concept throughout Topics 9-12.	**Note:** Students will need multiple opportunities to find equivalent fractions on a number line. Consider teaching this lesson over 2 days. Remember that by teaching this lesson over two days, it will be one of the F/D/E days on the WCSD Pacing Framework. **Solve & Share:** Consider having multiple tools readily available, like a ruler or Teaching Tool 12. Child-watch for students who may not know how to read a ruler and notice if students connect the ruler to a number line. **Look Back:** Consider facilitating a discussion around the *Look Back!* by having students find more fractions equivalent to 2/4 by using the ruler or number line. Students may also want to use fraction strips or an area model to find equivalent fractions. **Visual Learning:** Consider having students "act out" the number line and decide where fractions may fall on the number line to show equivalence. For example, some students may hold the following numbers on a number line; 0, ¼, 2/4, ¾ and 1. Other students will add the eighths onto the already made number line. Continue with twelfths. This reinforces the idea that equivalent fractions must be part of the same-sized whole. Consider using a variety of number line lengths for students to see the number line goes beyond 1. **Convince Me:** Consider using the *Convince Me!* to facilitate a discussion regarding the question. Have students connect to the *Visual Learning Animation*. **Another Example:** Consider having a discussion with the whole class about the *Another Example!*. Consider making a number line that is greater than 1 and have students "act out" where equivalent fractions would fall on the already made greater than 1 number line. ***-continues on next page-***

		Consider utilizing this question format during practice: Example 2 Full Statement **Example Stem 2:** Figure B shows several number lines that divide 1 into equal parts.  Enter another fraction that is equal to $\frac{4}{12}$.
Lesson 8-3: Generate Equivalent Fractions- Multiplication		
4.NF.A.1 MP.2 **MP.3** MP.4	**Access Prior Learning**: In third grade, students used the Identity Property of Multiplication for whole numbers. In the previous lessons, students used area models and number lines to find equivalent fractions. **Beginning of the Big Idea:** In this lesson, students will work with equivalent fractions by applying the Identity Property of Multiplication to develop a procedure for finding equivalency between fractions.	**Note:** Focus on why fractions are equivalent when multiplying by the same nonzero fraction. The Identity Property of Multiplication is explained in *Essential Question* (TE, p. 424). **Solve & Share:** Consider giving students opportunities to use tools and models to solve the problem. Students can use two colored counters to solve the set model problem. **Look Back:** Consider facilitating a discussion around the *Look Back!* to see if students make connections between the numerators and denominators of the equivalent fractions. If students do not make connections, consider using this lesson to continue with a conceptual understanding of equivalent fractions through representations before showing the procedure. **Visual Learning:** In the *Visual Learning*, students are shown multiplication to find equivalent fractions. Consider reading the questions and *Prevent Misconception* as there is important information and questions to ask students regarding multiplication to find equivalent fractions (TE p. 424). Reiterate the relationship between the numerator and denominator and not the size. **Convince Me:** Consider facilitating a discussion around the *Convince Me!* as it connects to the *Visual Learning Animation.* **Guided Practice:** Consider facilitating a discussion around item 1 with the whole class as it compares to the area model with multiplication to find equivalent fractions. Consider giving students an opportunity to use area models, number lines or fraction strips to compare 5/6 and 10/12. **Independent Practice/Math Practices and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. **Assess and Differentiate/Intervention Activity:** Consider using area models, number lines or two colored counters for the *Intervention Activity* to support the conceptual understanding of equivalent fractions.

Lesson 8-4: Generate Equivalent Fractions- Division

4.NF.A.1

MP.1
MP.2
MP.3
MP.4
MP.6

Access Prior Learning:
In Topic 7, students learned to find factors of whole numbers. In the previous lesson, students learned to find equivalent fractions by multiplying the numerator and denominator by the same whole number greater than 1.

Beginning of the Big Idea:
In this lesson, students will learn to find equivalent fractions by dividing the numerator and denominator by a number that is a factor of both.

Note: If students are still working conceptually with equivalent fractions, consider using this opportunity to facilitate discussions around using models when comparing equivalent fractions.

Solve & Share:
Consider giving students the opportunity to use multiple tools or models to find equivalent fractions for 6/10.

Visual Learning:
The mathematical vocabulary of common factor is discussed in the *Visual Learning Animation*. Students find common factors greater than 1 by which to divide the numerator and denominator by. This idea begins to support simplifying fractions, which is still finding an equivalent fraction.

The *Visual Learning Animation* uses denominators beyond those indicated in the standards. Only use if these ideas do not come out during the whole class discussion following the Solve and Share. Ensure student engagement and understanding by stopping the animation and discussing key ideas.

Convince Me:
Consider facilitating a discussion around the *Convince Me!*, as it connects the division learned in the *Visual Learning Animation* to a number line used to find equivalent fractions in Lesson 8-2.

Independent Practice/Math Practices and Problem Solving:
Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider giving students multiple opportunities to use various strategies, tools and models to complete the problems.

Assess and Differentiate/Intervention Activity:
Consider using the *Intervention Activity* with students, but give students who are having difficulty with equivalent fractions an opportunity to use tools and models.

Consider utilizing this question format during practice:

Example 1
Full Statement

Example Stem: Select True if the equation is true. Select False if the equation is **not** true.

	True	False
$\frac{4}{6} = \frac{8}{12}$		
$\frac{50}{100} = \frac{3}{4}$		
$\frac{6}{8} = \frac{75}{100}$		

Lesson 8-5: Use Benchmarks to Compare Fractions

4.NF.A.2
4.NF.A.1

MP.1
MP.2
MP.3
MP.8

Access Prior Learning:
In third grade, students learned how to compare fractions with the same numerator or denominator by reasoning about the size of the fractions.

Developing the Big Idea:
In this lesson, students will learn about benchmark fractions and how benchmark fractions are used to compare fractions.

Solve & Share:
Consider handing students strips of paper all the same length, and having students shade different portions of each strip. Then have students estimate how much of the strip is shaded, and explain how they estimated. Consider having students critique the reasoning of others or construct an argument of why a student may agree or disagree with another student based on the estimate. Look for students who use benchmark fractions to compare.

Look Back:
Consider facilitating a discussion around the *Look Back!*, as this will support students thinking in regards to comparing fractions to ½, and the relationship between the numerator and denominator.

Visual Learning:
In the *Visual Learning Animation*, compare to ½ to determine equality or inequality.

.-continues on next page-

	Look Ahead: Benchmark fractions will be an important part of estimating with unlike denominators in Topic 9. Continue to use estimation language and emphasize accurate when comparing to benchmark fractions throughout Topic 8.	**Convince Me:** In the *Convince Me!*, students use a model or number line to compare fractions. Consider facilitating a discussion around the *Convince Me!*. **Another Example:** Consider facilitating a discussion around the *Another Example!* as students use one whole as a benchmark to compare fractions. **Assess and Differentiate/Intervention Activity:** In the *Intervention Activity,* students can solve the activity using fractions strips, Teaching Tool 13 or student-made strips from Lesson 8-1 to continue understanding fractions conceptually.
Lesson 8-6: Compare Fractions		
4.NF.A.2 **4.NF.A.1** MP.2 **MP.3** MP.5	**Access Prior Learning**: In third grade, students learned how to compare fractions with the same denominator by comparing numerators. **Developing the Big Idea:** In this lesson, students will compare fractions by changing them to equivalent fractions with a common denominator or numerator.	**Solve & Share:** Consider giving students opportunity to use various strategies, tools or models to compare fractions in the *Solve & Share*. Consider child-watching for misconceptions because they may think larger numbers means a larger fraction. Reiterate the relationship between the numerator, denominator and relative size. **Look Back:** Consider using the *Look Back!* as an extension for early finishers or use the *Look Back!* to facilitate a discussion around comparing fractions using inequalities. **Visual Learning:** In the *Visual Learning Animation*, students use their prior knowledge of multiples to find common denominators to compare fractions. Consider facilitating a discussion around why this procedure works in comparing fractions conceptually. Number lines are a model used to determine the equality and inequalities of fractions. **Another Example:** Consider facilitating a discussion around the *Another Example!* as students apply the knowledge of equivalent fractions to compare numerators. The *Another Example!* gives students another strategy to use when comparing fractions. **Independent Practice/Math Practices and Problem Solving:** Consider having students do items 6-15. Consider having students explain how they compared the fractions and why it may be correct. Child-watch to see if students apply number sense using tools, strategies or models to solve the items. Consider facilitating a discussion around item 16 with the whole class, as students need to understand when comparing fractions, the fractions need to represent the same whole. Concepts of area could be used to explain why the fractions are not equivalent. **Assess and Differentiate/Intervention Activity:** Consider facilitating a discussion with all students around the *Intervention Activity* as students compare fractions with like numerators, with like denominators or neither. Consider modifying a question to include the following example format during practice: Example 1 Full Statement **Example Stem:** Select the symbol (< , > , or =) that correctly compares each pair of numbers. |	< | > | = | | $\frac{2}{8} \square \frac{1}{4}$ | | | | | $\frac{3}{5} \square \frac{7}{8}$ | | | | ***CTC: *Quick Check*** (digital platform)

Lesson 8-7: Math Practices and Problem Solving- Construct Arguments		
4.NF.A.1 **4.NF.A.2** **MP.3** MP.1 MP.2 MP.5	**Access Prior Learning**: In previous topics and lessons, students have constructed arguments. **Developing the Big Idea:** In this lesson, students will construct arguments about comparing fractional amounts.	**Note:** This lesson focus is on constructing arguments in math, but the mathematical idea is to compare fractions based on the same-sized whole. **Solve & Share**, **Visual Learning** and **Convince Me** all focus on the mathematical idea when comparing fractions the comparison needs be part of the same-sized whole. **Independent Practice/Math Practices and Problem Solving:** Consider having students do items 6-9, as students need to understand the values of the fractions and where the fractions fall on a number line. Similar to Van de Walle, et al. (2010) problem "Who is Winning?" (p. 290).

References

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Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

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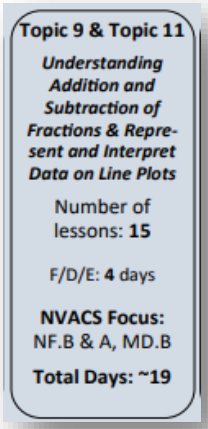
Grade 4 Topic 9: Understand Addition and Subtraction of Fractions



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: Number and Operations- Fractions (pp. 121-125)

Prior to instruction, view the Topic 9 Professional Development Animation located in Pearson Realize online). Read the Teacher Edition (TE), Cluster Overview/Math Background (pp. 461A-461F), the Topic Planner (pp. 461I-461K), all 11 lessons, and the Topic Assessment (pp. 537-538A).

Mathematical Background: Read Cluster Overview (TE, pp. 461A-461F)	**Topic Essential Questions:** How can we use estimation to determine the reasonableness of sums and differences when adding/subtracting fractions referring to the same whole? How do you add and subtract fractions and mixed numbers with like denominators? How can fractions be added and subtracted on a number line? *Reference TE p. 461 and Answering the Topic Essential Questions (TE, pp. 535-536) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

9-7	9-1	9-2	9-3	9-4	9-5	9-6	9-8	11-1	9-9	11-2	9-10	11-3	9-11	11-4	Assessment

4 F/D/E days used strategically throughout the topic

Instructional note:

Topics 9 and 11 are combined to make connections with fractions on a number line. In these topics, students are finding differences of values on the line plot, as well as making number lines to graph the given data. Consider teaching lessons 9-7, 9-1 to 9-8, and then alternate between the two topics: 11-1, 9-9, 11-2, 9-10, 11-3, 9-11 and 11-4. Be aware lesson 9-7 is taught before 9-1, as students estimate fraction sums and differences with unlike denominators. Lesson 9-7 connects to the work done around equivalent fractions in Topic 8. This was field tested in four (4) WCSD Title I schools during the 2017/2018 school year, and found to better support conceptual understanding and concept development.

Topic 9 is the beginning of a cluster for standard 4.NF.B, "build fractions from unit fractions by applying and extending previous understanding of operations of whole numbers" (Nevada Academic Content Standards (NVACS), 2010). Focus instruction on 4.NF.B.3.A-D. In the standards, students will "add and subtract fractions and mixed numbers with like denominators by understanding addition and subtraction as joining and separating parts referring to the same whole. Decompose fractions with the same denominator in more than one way. Add and subtract fractions by replacing each mixed number with an equivalent fraction, use the properties of operation and the relationship between addition and subtraction by using visual models and equations to represent the problem" (NVACS, 2010, 4.NF.B).

"The idea that the top number (numerator) counts and the bottom number (denominator) tells what is counted makes addition and subtraction of like fractions the same as adding and subtracting whole numbers" (Van de Walle, Karp, & Bay-Williams, 2010, p. 315). A key idea about fractions that students must come to understand is that a fraction does not say anything about the size of the whole or the size of the parts. A fraction tells us only the *relationship between* the part and the whole" (Van de Walle, et al., 2010, p. 288). When students add and subtract fractions, they need to consider that both fractions are part of the same whole.

Small identifies common misconceptions about adding fractions, "because of students' greater comfort with whole numbers than fractions, when they see a calculation such as 2/3 + 1/3, it is natural for them to add numerators and add denominators to get 3/6. Hopefully, if students have been encouraged to estimate, they would realize that it is impossible to start with more than ½ (which 2/3 is), add a positive amount, and end up with 3/6, which is ½" (2014, p. 51). "The development of fraction number sense should most certainly include estimation of fraction sums and differences. In most cases students' estimates should not be much more than ½ away from the exact sum or difference" (Van de Walle, et al., 2010, p. 311).

Models are important when students are beginning to understand fractional concepts. The different models give students various opportunities to learn fractions. Please see the *Instructional Note* in WCSD Topic 8 Curriculum Guide for more information on the different types of models. Understanding that concrete tools are important as students begin their work with adding and subtracting fractions will support them throughout the topic.

Focus Math Practice 4: Model with math

Focus on opportunities for students to develop *Mathematical Practice 4* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 9-10. Reference the Teacher's Edition (pp. F24-F24A) and the NVACS (2010, p. 7).

Grade 4 Topic 11: Represent and Interpret Data on Line Plots

Big Conceptual Idea: Measurement and Data (Data Part) (pp. 66-68)

Prior to instruction, view the Topic 11 Professional Development Video (located in Pearson Realize online). Read the Teacher Edition (TE), Cluster Overview/Math Background (pp. 587A-587F), the Topic Planner (pp. 587I-587J), all 4 lessons, and the Topic Assessments (pp. 621-622A).

Mathematical Background:	**Topic Essential Questions:**
Read Cluster Overview- (TE, pp. 587A-587F)	How can you read data on a line plot? How can you make a line plot? *Reference TE (p. 587) and Answering the Topic Essential Questions (TE, pp. 619-620) for key elements of answers to the Essential Questions.*

Instructional note:

Topic 11 focuses on representing and interpreting line plots. Focus for standard MD.B.4, is to make a line plot to display a data set of measurement in fractions of a unit (1/2, ¼, 1/8) (Nevada Academic Content Standards (NVACS), 2010). "**Mathematics** is about numbers and their operations, generalizations and abstractions; it is about spatial configurations and their measurements, transformations and abstractions. **Statistics** is also about numbers-but numbers in context; these are called data. Statistics is about variables and cases, distribution and variation, purposeful design or studies, and role of randomness in the design of studies and interpretation of results" (Van de Walle, Karp, Lovin, & Bay Williams, 2014, p. 378).

There are two main types of data: categorical and numerical. Kindergartners begin to work with categorical data. This continues until second grade when both categorical and numerical data is used. Beginning in second grade, students may begin "discussing and interpreting visual features of line plots, such as '**outlier'** value" (Progression Document for Common Core Math Standards, Measurement and Data, pg. 10).

Throughout Topic 11, students interpret the data presented in the line plot. "Line plots or dot plots count things along a numerical scale. The number line is drawn and an X or • is made above the corresponding data elements. The advantage of a line/dot plot is that every piece of data is shown on the graph" (Van de Walle, Karp, Bay-Williams, 2010, p. 446). One way students will interpret the line plot is by looking for outliers. An outlier is any number in the data set that is very different from the rest of the numbers. Not only do students look for outliers, interpretation of a line plot can also be represented by the "most common" values on the given line plot. Students can also look for gaps and cluster on the line plot and interpret the reason for a gap or cluster (often times referenced as the 'shape' of the data).

Focus Math Practice 3: Critique reasoning

Focus on opportunities for students to develop *Mathematical Practice 3* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 11-4. Reference the Teacher's Edition (pp. F23-F23A) and the NVACS (2010, p. 7).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Topics 9 & 11 Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
decompose compose mixed fraction line plot outlier	*fraction* *numerator* *denominator* *reasonable* *equivalent fraction* *Associative Property of Addition* *Commutative Property of Addition* *data* *scale*

***Additional terminology that students may need support with*:** equations, whole, part of the whole, expression, rename, relationship, most often, gaps and clusters

Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "Are students able to add or subtract fractions with like denominators using tools, strategies or models?"
"Are students able to create and interpret a line plot based on the given data?"

Lesson	Evidence	Look for
9-5	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • students tools, strategies and models. • students decomposing and reasoning about the size of the whole.
11-3	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • students creating an accurate number line and line plot. • students interpreting the line plot correctly based on the information represented.

Learning Cycle Assessments (summative)	***Topic 9 & 11 Performance Assessments*** SE pp. 537-538 and SE pp. 622	Use *Scoring Guide* TE pp. 537-538A and 622

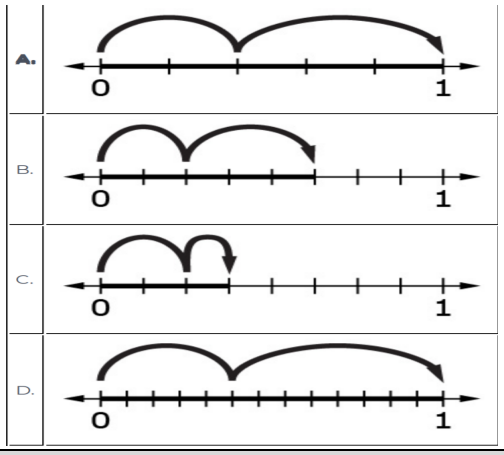
Standards listed in **bold** indicate a focus of the lesson.

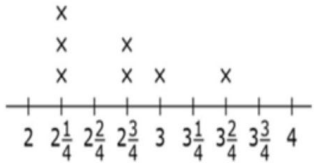
NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 9-7: Estimate Fraction Sums and Differences		
4.NF.B.3a MP.1 MP.2 MP.3 **MP.4** MP.8	**Access Prior Learning**: In previous lessons, students added and subtracted fractions using various tools and representations. **Beginning of the Big Idea:** In this lesson, students will estimate sums and differences by replacing fractions with benchmark fractions that are close and easy to add or subtract.	**Note:** In this lesson, students will estimate fractions with different denominators. Consider doing this lesson before 9-1 as it reinforces concepts in Topic 8. 9-7 builds an understanding of benchmark fractions to estimate sums and differences as well as uses the number line to show equivalent fractions in order to add and subtract fractions with like denominators. **Solve & Share:** Consider using the *Solve & Share* as a Number Talk. After students mentally decide if the expressions are greater or less than one (problems 1-2), have them explain how they know. Repeat the same with problems 3-4, except have students decide if the expressions are less than or greater than ½. Consider recording students' generalization in regard to estimation. **Visual Learning:** Consider reading *Prevent Misconceptions* prior to lesson, to guide students as they work through the *Visual Learning* (TE, p. 502). The *Visual Learning Animation* guides students to understand they can rename a whole number to an equivalent fraction. **Convince Me:** Consider using the *Convince Me!* formatively to assess student understanding that fractions must refer to the same whole before they can be compared, added or subtracted. This is important for students to show an understanding of fractions when comparing, adding or subtracting.
Lesson 9-1: Model Addition of Fractions		
4.NF.B.3a MP.1 MP.2 MP.3 **MP.4** MP.5	**Access Prior Learning**: In third grade, students developed an understanding of the meaning of addition of whole numbers as joining and they developed an understanding of the meaning of a fraction a/b as a number of unit fractions 1/b. **Beginning of the Big Idea:** In this lesson, ideas learned in the previous grades are brought together through representations to show how fractions with the same denominator can be joined.	**Solve & Share:** Consider giving students the opportunity to use tools and other representations to solve the *Solve & Share* problem. **Look Back:** Consider facilitating a discussion around the *Look Back!* as students begin to understand conceptually how to add fractions with like denominators through concrete tools. **Visual Learning:** Consider reading the *Prevent Misconception* to support students as they add like denominators using fraction strips (length models) and a number line model (TE, p. 466). ***-continues on next page-***

		Along with using the fraction strips and making a number line presented in the book, consider encouraging students to use two-colored counters, as the *Visual Learning* problem is fractions of a set (set model). **Note:** Students may struggle with the concept of "either", as well as using the fraction strips to model the context. **Guided Practice:** Consider facilitating a discussion around items 1-3 as students continue to develop a conceptual understanding with adding fractions with like denominators. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students, as students use a real-world context to add fractions with like denominators.
Lesson 9-2: Decompose Fractions		
4.NF.B.3b MP.2 **MP.4** MP.5	**Access Prior Learning**: In third grade, students used number lines to find simple equivalent fractions and to explain why they are equivalent. **Developing the Big Idea:** In this lesson, students continue to find equivalent fractions, by using the number line and area models.	**Solve & Share:** Consider making an anchor chart of students' decomposition of 11/8 pounds into three bowls. Students will see that there are many ways to decompose fractions when adding. **Look Back:** Consider facilitating a discussion around the *Look Back!* as students revisit equivalent fractions. **Visual Learning:** The mathematical terminology of decompose and compose fractions is discussed in the *Visual Learning Animation*. Consider having students complete the problem before showing the animation to make connections with their answer. **Convince Me:** Consider using the *Convince Me!* to facilitate a discussion as students continue to connect addition of fractions to the concrete tool, such as fraction strips. **Another Example:** Consider having a discussion with the whole class about the *Another Example!* as students decompose a fraction greater than 1 (mixed number). Consider calling a mixed number a fraction greater than one also and not just a mixed number. This supports students' knowledge of how fractions can have whole numbers and part of the whole. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students as this may introduce fraction circles in a different way than students have seen before.
Lesson 9-3: Add Fractions with Like Denominators		
4.NF.B.3a **4.NF.B.3d** MP.1 MP.3 **MP.4** MP.7	**Access Prior Learning**: In previous lessons, students connected the meaning of fraction addition to whole number addition, and that a fraction greater than 1 can be decomposed different ways. **Developing the Big Idea:** In this lesson, students will continue adding fractions with like denominators by decomposing the fraction into unit fractions and then joining them together to find the total.	**Solve & Share:** Consider giving students the opportunity to use tools and models to solve the problem and removing the fraction strip model to elicit more students' strategies. **Look Back:** Consider facilitating a discussion around the *Look Back!* to see if students make connections between the denominators in the equation. **Visual Learning:** In the *Visual Learning*, students solve the problem by using a number line model. Consider having students make their own number line in their math journal. Consider emphasizing the idea that the denominators represent the same whole, and this is why students can add fractions. **Convince Me:** Consider facilitating a discussion around the *Convince Me!* as students construct an argument around Frank's error. **Another Example:** Consider having a discussion around the *Another Example!*, as students write the fraction greater than one as a mixed number. Discuss how students know when a whole number is formed. ***-continues on next page-***

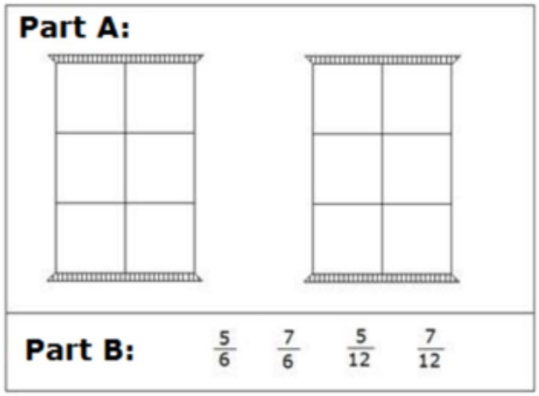
		Guided Practice: Child-watch for students who may reason through item 1, *Guided Practice*, by using the table from the *Visual Learning Animation*. Consider utilizing this question format during practice: Example 2 Full Statement **Example Stem 2:** Enter the unknown number that makes the equation true. $\frac{4}{5} = \square + \frac{2}{5}$
Lesson 9-4: Model Subtraction of Fractions		
4.NF.B.3a MP.1 MP.2 **MP.4** MP.5 MP.6	**Access Prior Learning**: In the previous lesson, students developed the idea that a fraction a/b, where a > 1, can be decomposed into unit fractions, 1/b. **Beginning of the Big Idea:** In this lesson, students will subtract fractions a/b – c/b, where a > 1, is given meaning by first decomposing a/b into unit fractions, 1/b, and then separating c unit fractions from the total.	**Solve & Share:** Consider having multiple tools readily available and giving students an opportunity to represent this problem by asking students to try multiple ways to solve the problem. **Visual Learning:** Consider reading the *Prevent Misconceptions* questions regarding subtracting fractions and apply these during the *Visual Learning Animation* (TE, p. 484). Students write one whole as a fraction to subtract fractions. Use this opportunity to review that fractions are equal parts and not necessarily rectangle. **Another Example:** The *Another Example!* builds upon the *Visual Learning Animation* and has students write the difference as a fraction or number greater than one (mixed number) using fraction strips. **Another Look! On Homework & Practice pg. 487** The *Another Look!* represents the subtraction of fractions using another strategy: Fraction Circles. **Independent Practice/Math Practices and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider giving students multiple opportunities to use various strategies, tools and models to complete the problems. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students but give students who are having difficulty with subtraction of fractions an opportunity to use tools and models. Consider utilizing this question format (based on where the equal sign is being placed) during practice: Example 2 Full Statement **Example Stem 2:** Enter the unknown number that makes the equation true. $\square = \frac{4}{8} - \frac{1}{8}$
Lesson 9-5: Subtract Fractions with Like Denominators		
4.NF.B.3a **4.NF.B.3d** MP.2 MP.3 **MP.4**	**Access Prior Learning**: In the previous lesson, students subtracted fractions with like denominators by decomposing the minuend into the sum of two fractions and then separating from that the subtrahend (the second number).	**Solve & Share:** Consider looking for students who use a number line as the context deals with distance. ***-continues on next page-***

	Developing the Big Idea: In this lesson, students will use the inverse relationship between addition and subtraction to find the difference of two fractions with like denominators.	**Visual Learning:** Consider reading *Prevent Misconceptions* to make connections between adding and subtracting fractions with like denominators (TE, p. 490). In the *Visual Learning*, students compare a number line model and a bar diagram to solve the problem. Consider emphasizing that the denominators represent our whole or unit, so they stay the same while subtracting. Remind students about equations and have them write the equation that goes with the problem. **Guided Practice:** In the *Guided Practice*, item 1 students review equivalent fractions. **Independent Practice/Math Practices and Problem Solving:** For item 21, students use a set model to subtract. Set models are sets of objects understood to be the whole and each subset makes up fractional parts of the whole (Van de Walle, et. al, 2010, p. 290). **Assess and Differentiate/Intervention Activity:** Consider facilitating a discussion around the *Intervention Activity* as the activity reviews the meaning of the numerator and denominator. Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem 1:** Enter the unknown number that makes the equation true. $\frac{7}{5} - \square = \frac{4}{5}$ ***CTC:** ***Solve & Share*** (student work samples)
Lesson 9-6: Add and Subtract Fractions with Like Denominators		
4.NF.B.3a MP.2 **MP.4** MP.5	**Access Prior Learning**: In this topic, students have added and subtracted fractions with like denominators by joining segments or by separating segments on a number line. **Developing the Big Idea:** In this lesson, students continue adding and subtracting like fractions by extending previous knowledge to counting forward or counting backward on a number line depending on the operation. **Look Ahead:** Emphasize the number line strategy, as it will be used in Topics 10-12.	**Solve & Share:** Consider giving students opportunity to use various strategies, tools or models to solve the problem. Consider removing the "Think Bubble" as it gives students the idea of using the number line. Look for students who use the number line on their own. **Look Back:** Consider using the *Look Back!* as it revisits equivalent fractions. **Visual Learning:** In the *Visual Learning*, students use a number line to add like fractions by using jumps on the number line. Consider giving students opportunity to solve the problem and share how they solved it. Look at what derived facts, or known facts, students used to help them make the jumps on the number line. **Note:** Give students the opportunity to use different size jumps instead of one whole jump. For example, students may need to jump one by one, two by two, etc. **Another Example:** The *Another Example!* asks students to solve a subtraction problem by using a number line and counting backwards. Again, consider giving students opportunity to solve the problem and share how they solved it. Look at what derived facts, or known facts, students used to help them make the jumps on the number line. **Independent Practice/Math Practices and Problem Solving:** For item 11, have students estimate before solving the problem. ***-continues on next page-***

		Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** Select the model that matches this equation. $\frac{5}{8} = \frac{2}{8} + \frac{3}{8}$ 
Lesson 9-8: Model Addition and Subtraction of Mixed Numbers		
4.NF.B.3c MP.2 MP.3 MP.5	**Access Prior Learning**: In previous topics and lessons, students have constructed arguments. **Developing the Big Idea:** In this lesson, students will construct arguments about comparing fractional amounts.	**Look Back:** Consider having students answer the *Look Back!* before they work on the *Solve & Share* as it asks them to estimate the *Solve & Share*. **Solve & Share:** Consider giving students the opportunity to use tools or representations to solve the problem. Consider having students solve the problem in a couple of different ways. **Visual Learning:** Students use fraction strips and a number line to solve the problems. Consider giving students opportunity to solve the problem before showing the animation or having a discussion in regards to the *Visual Learning*. Consider reading the *Prevent Misconceptions* prior to teaching the lesson (TE, p. 508).
Lesson 11-1: Read Line Plots		
4.MD.B.4 MP.2 **MP.3** MP.6 MP.7	**Access Prior Learning**: In third grade, students recorded data on line plots marked with half and quarter inches. **Developing the Big Idea:** In this lesson, students extend their previous understanding to read line plots with fractional units.	**Note:** Throughout the topic, dots are used instead of X's. Consider showing students the different representations for numbers on the line plot. **Look Back:** Consider facilitating a discussion around the *Look Back!* as students begin to think about relationships on the line plot. Connect back to information discussed in the *Solve & Share*. **Visual Learning:** In the *Visual Learning Animation*, mathematical terminology for line plots is introduced. Consider facilitating a discussion around the information in the table and how this information is transferred to the line plot. **Convince Me:** Consider facilitating a conversation around the *Convince Me!* as students interpret the data set with focus on the outlier as well as what the outlier represents on the line plot. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students. Students learn how to collect data and use the data collected to represent it on a line plot. Consider having students decide what numerical data to collect. Consider posing the questions from the *Intervention Activity* for students to think about and answer regarding the information they collected. ***-continues on next page-***

		Consider modifying a question to include the following example format: Example 1 Full Statement **Example Stem:** A student measured how much rain fell each week. This line plot shows the amount of rain, in inches, that fell each week.  **Amount of Rain That Fell Each Week (in)** How much more rain, in inches, was there during the week with the greatest amount of rain than during the week with the least amount of rain? Enter your answer in the response box.
Lesson 9-9: Add Mixed Numbers		
4.NF.B.3c MP.1 MP.2 MP.3 MP.8	**Access Prior Learning**: In the previous lesson, students used fraction strips and number lines to add mixed numbers, or fractions greater than one. **Developing the Big Idea:** In this lesson, students will learn two procedures for adding mixed numbers, or fractions greater than one.	**Look Back:** Consider having students do the *Look Back!* before finding the solution to the *Solve & Share*, as it has students estimating the sum. **Solve & Share:** Consider giving students the opportunity to use tools, such as actual measuring cups or representations to solve the problem. Consider having students solve the problem in a couple of different ways. **Visual Learning:** In the *Visual Learning*, students learn to use the properties of operations (Commutative and Associative Property) to add mixed numbers. Students use the strategies of breaking apart the mixed number to add or using equivalent fractions to add. Consider having students work out each way during the discussion around the *Visual Learning*. Consider having students compare the different strategies in the *Visual Learning*. **Convince Me:** Connect the *Convince Me!* to the discussion during the *Visual Learning Animation*. **Independent Practice/Math Practices and Problem Solving:** For item 23, have students estimate before solving the problem.
Lesson 11-2: Make Line Plots		
4.MD.B.4 **4.NF.A.1** MP.2 **MP.3** MP.6	**Access Prior Learning**: In previous grades, students have collected measurement data and displayed it using a line plot. **Developing the Big Idea:** In this lesson, students will continue interpreting the line plots they make by displaying the information from the data collected.	(**Possible 2-day lesson)** **Note:** If this lesson is extended over 2-days, this will be one of the F/D/E days given for this topic on the WCSD Pacing Framework. **Day 1:** **Solve & Share:** Consider using the data given in the *Solve & Share*, or consider collecting data on the number of pets students have in the class. Have students use the information collected to make a line plot. Represent the whole numbers with either a dot or X. This line plot uses whole numbers, as this may be the first experience with making a line plot. **Look Back:** Consider facilitating a discussion around the *Look Back!* and the data collected in the *Solve & Share* to discuss the data that occurs "most often" or mode. **Day 2:** **Visual Learning:** Read the questions and *Prevent Misconceptions* before teaching the lesson (TE, p. 598). Consider using the table to have students make a line plot to represent the fractional data in the table. Consider using the questions given to facilitate a discussion around the *Visual Learning Animation*. ***-continues on next page-***

		Convince Me: Consider having students complete the *Convince Me!* as a Gallery Walk (ELL Toolkit p. 22) to make the line plot with the shoe sizes. Have students generate their own questions regarding their line plot for students to answer as they visit the poster.
Lesson 9-10: Subtract Mixed Numbers		
4.NF.B.3c MP.1 MP.2 MP.3 MP.8	**Access Prior Learning**: In the previous lesson, students used fraction strips and number lines to subtract mixed numbers, or fractions greater than one. **Developing the Big Idea:** In this lesson, students will learn three procedures for subtracting mixed numbers, or fractions greater than one.	**Solve & Share:** Consider removing the bar diagram to elicit more student strategies. If students seem to be struggling, guide them through questions towards the bar diagram. **Look Back:** Consider facilitating a discussion around the *Look Back!*, as students need multiple opportunities to estimate. Estimation is a good way for students to check for reasonableness of the answer. **Visual Learning:** In the *Visual Learning*, students learn to rename a fraction to be able to subtract without making a negative number. Students will also subtract by using equivalent fractions. Consider facilitating a discussion around the different procedures presented in the *Visual Learning Animation* to subtract mixed numbers. **Independent Practice/Math Practices and Problem Solving:** Consider giving students multiple opportunities to use various strategies, tools and models to complete the problems.
Lesson 11-3: Use Line Plots to Solve Problems		
4.MD.B.4 MP.1 MP.2 MP.5 MP.8	**Access Prior Learning**: In previous topics, students learned how to add and subtract mixed numbers. In the previous lessons, students learned to make and interpret line plots. **Developing the Big Idea:** In this lesson, students will read the data from given or made line plots, and use the data to solve problems that include adding or subtracting mixed numbers.	**Solve & Share:** In the *Solve & Share*, students have to find the difference between the longest and shortest caterpillar. Students are finding the range. **Visual Learning:** Read the *Prevent Misconceptions* before teaching the lesson (TE, p. 604). In the *Visual Learning*, students compare information between two line plots. Notice the line plots have the same numerical scale, consider facilitating a discussion around the numerical scales and how if they were different the line plots would look different (manipulating data). **Another Example:** In the *Another Example!*, students use the information in the line plots to decide the "most common or often" from the data. Students begin to think about averages or central tendencies. Also, consider asking students what the least common value is. **Assess and Differentiate/Intervention Activity:** Consider using the *Reteach Activity* with all students. This activity will reinforce how to find the intervals of fractions on a number line. ***CTC: *Solve & Share*** (student work samples)
Lesson 9-11: Math Practices and Problem Solving- Model with Math		
4.NF.B.3d **4.NF.B.3a** MP.1 MP.2 **MP.4** MP.5	**Access Prior Learning**: In previous topics and lessons, students have used Math Practice 4: Model with math. **Developing the Big Idea:** In this lesson, students will be able to develop good habits problem solvers use to model with math while adding and subtracting fractions with like denominators.	Throughout the lesson, students use bar diagrams to add and subtract fractions with like denominators. Consider giving students an opportunity to use the bar diagrams and write an equation that goes with the problem. Also, consider having students solve the problems using tools or other representations. **Assess and Differentiate/Intervention Activity:** Consider facilitating a discussion around the *Reteach Page* as students revisit variables and equations while applying this knowledge to the bar diagram. Consider utilizing this question format during practice: Example 2 Full Statement **Example Stem 2:** Michael and Erin have 2 bars of chocolate. Together they eat $1\frac{1}{6}$ bars of chocolate.  represents one bar of chocolate ***-continues on next page-***

		Part A: Shade the model to show the amount of chocolate they did **not** eat. **Part B:** Click on the fraction that shows the amount of chocolate they did **not** eat. 
Lesson 11-4: Math Practices and Problem Solving- Critique Reasoning		
4.MD.B.4 **MP.3** MP.1 MP.2 MP.4	**Access Prior Learning**: In previous grades, topics and lessons, students have used MP.3 to critique the reasoning of others. **Developing the Big Idea:** In this lesson, students critique the reasoning related to solving problems involving line plots.	**Visual Learning:** In the *Visual Learning*, students are adding fractions with like denominators based on the information provided by the two line plots. **Convince Me:** Consider facilitating a discussion around the *Convince Me!* as students critique Bev's reasoning based on the two line plots from the *Visual Learning Animation*. **Independent Practice/Math Practices and Problem Solving:** Consider having students work on items 6-8 as they create and interpret a line plot based on the information provided in the table.

References

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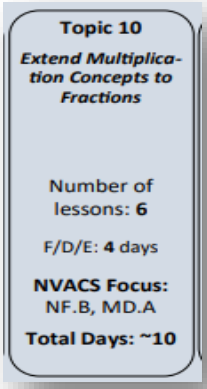
Grade 4 Topic 10: Extend Multiplication Concepts to Fractions



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: Number and Operations- Fractions (pp. 121-125)

Prior to instruction, view the Topic 10 Professional Development Video located in Pearson Realize online. Read the Teachers' Edition (TE): Cluster Overview/Math Background (pp. 461A-461F), the Topic Planner (pp. 539I-539K), all 6 lessons, and the Topic Assessments (pp. 585-586A).

Mathematical Background: Read Cluster Overview (TE, pp. 461A-461F)	**Topic Essential Questions:** How can you describe a fraction using a unit fraction? How can you multiply a whole number by a mixed number? *Reference TE (p. 539) and Answering the Topic Essential Questions (TE, pp. 583-584) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

10-1	10-2	10-3	10-4	10-6	10-5	Assessment

4 F/D/E days used strategically throughout the topic.

Instructional note:

This topic is the last of the cluster group for standard 4.NF.B, "build fractions from unit fractions by applying and extending previous understanding of operations of whole numbers" (Nevada Academic Content Standards (NVACS), 2010). Focus instruction on 4.NF.B.4, which is to apply and extend previous understanding of multiplication to multiply fractions by a whole number. Students "first experience with multiplication of fractions should involve finding fractions of whole numbers" (Van de Walle, Karp, Bay-Williams, 2010, p. 317).

In this topic, students will extend their understanding of whole number multiplication to include multiplying whole numbers by fractions. Students recognize multiplication expressions such as 3 x 7 as 3 groups of 7 objects or as repeated addition; 7 + 7 + 7. Building on these understandings and including iteration of the unit fraction, students learn that a fraction such as ¾ can also be represented as 3 x ¼. As stated in enVisionmath2.0 Teacher's Edition, "Students come to understand that they can think of a fraction as the product of a whole number and a unit fraction" (p. 461D). A unit fraction describes one part of the whole, with the numerator always being one. This means that 5 x ¾ can also be represented as 5 x (3 x ¼). Using the associative property, the expression can also be represented as (5 x 3) x ¼, allowing students the flexibility to multiply whole numbers and then partition the whole by the denominator of the unit fraction. This also helps build understanding of how multiplying by a fraction creates a product that is less than one of the factors.

Models are important when students are beginning to understand fractional concepts. The different models give students various opportunities to learn fractions. These different models include; region or area, length or number line and set models. For example, "an area model helps students visualize parts of the whole. A linear model shows that there is always another fraction to be found between any two fractions-an important concept that is underemphasized in the teaching of fractions" (Van de Walle, et al., 2010, p. 288). Van de Walle, et al. (2010), continues to emphasize the use of models, "It is important to remember that students must be able to explore fractions across models. If they never see fractions represented by length, they will struggle to solve any problem or context that is linear. As a teacher you will not know if they really understand the meaning of fractions unless they can model a fraction using different context or models" (pp. 290-291).

"Students can also use the distributive property and find the partial products, just as they do when multiplying two-digit whole numbers. The process is more conceptual and also lends itself to estimation, either before the partial products are determined or after" (Van de Walle, et al., 2014, pp. 246-247). "Misconceptions can be intensified when students are too quickly pressed to memorize rules, such as "multiply both the top and bottom" and are not given adequate time to explore multiplication of fractions conceptually" (Van de Walle, et al., 2014, p. 247). Van de Walle, et al., go on to list a variety of misconceptions students develop about multiplication of fractions:

- treating the denominator the same as in addition/subtraction problems
- inability to estimate approximate size of the answer due to the idea that "multiplication makes things bigger"
- matching multiplication situations with multiplication, not division
- using key word strategies

(2014, p. 247)

Math Practice 4: **Use tools strategically**

Focus opportunities for students to develop *Mathematical Practice 4* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 10-6. Reference the Teacher's Edition (pp. F24-F24A) and the NVACS (2010, p. 7).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
	unit fraction *mixed number*

Additional terminology that students may need support with: equations, whole, part of the whole, expression, distributive property, partial products, time, area model or open array

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students developing conceptual understanding and moving their thinking from less sophisticated understandings (fraction strips) toward equations through use of repeated addition or multiplication?"

Lesson	Evidence	Look for
10-3	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • student strategies and models. • use of repeated addition of the unit fraction or fraction. • use of multiplication of a whole number by a fraction or unit fraction.
10-6	***Quick Check*** (digital platform)	Focus CTC around data analysis and collection of student workspace (scratch paper). Printable version available under "Teacher Resources".

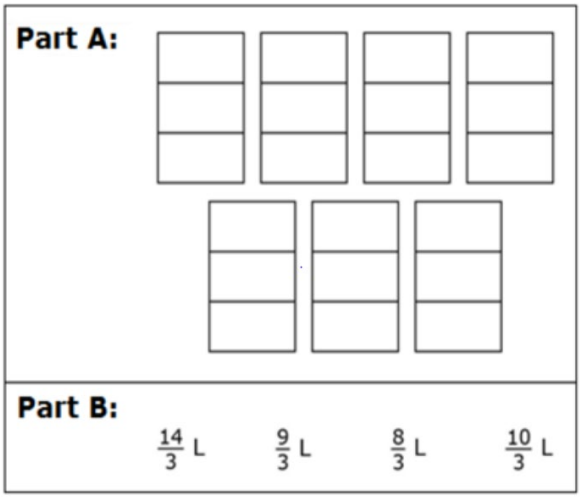
Learning Cycle Assessments (summative)	***Topic Performance Assessment*** SE pp. 585-586	Use *Scoring Guide* TE pp. 585-586A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 10-1: Fractions as Multiples of Unit Fractions- Use Models		
4.NF.B.4a MP.2 **MP.4** MP.7	**Access Prior Learning**: In third grade, students learned the meaning of multiplication, including as repeated addition. In the previous topic, students learned to represent a fraction a/b as a sum of the fractions. **Beginning of the Big Idea:** In this lesson, students combine these understandings to write a fraction a/b as a product of a x 1/b.	**Solve & Share:** Consider changing the *Solve & Share* and instead use item 19 from *Math Practices and Problem Solving* to elicit more student strategies and to increase the cognitive demand. Item 19 states, "Mark is slicing tomatoes for 4 members of his family. Each person will get 1/6 of the tomato. What fraction of the tomato will Mark slice?" Consider connecting the *Solve & Share* to item 19, after students have had opportunity to work on the problem, shared strategies and a discussion has taken place. **Visual Learning:** The mathematics terminology, unit fraction, is discussed in the *Visual Learning*. Students use the unit fraction to show multiplication as repeated addition. Students make connections to previous grade and topics to solve fraction multiplication problems. Consider reading the *Prevent Misconception* prior to the lesson to support students as they multiply fractions (Teacher's Edition (T.E.), p. 544). **Convince Me:** Consider facilitating a discussion around the *Convince Me!* to connect ideas in this lesson to ideas in Topic 9 related to the parts of the fractions and their meaning. **Another Example:** The *Another Example!* connects unit fractions to a fraction greater than one. Consider giving students opportunity to use tools as a discussion is facilitated around the problem. Connect the *Another Example!* to the *Visual Learning Animation*. ***-continues on next page-***

		Independent Practice/Math Practices and Problem Solving: Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider giving students multiple opportunities to use various strategies, tools and models to complete the problems to extend their understanding. Consider modifying a question to include the following phrase: **Example Stem:** Enter the unknown number that makes the equation true. $\square = 4 \times \frac{1}{12}$
Lesson 10-2: Multiply a Fraction By a Whole Number- Use Models		
4.NF.B.4b **4.NF.B.4a** **4.NF.B.4c** MP.2 **MP.4** MP.7 MP.8	**Access Prior Learning**: In the previous lesson, students learned to write a fraction as the product of a whole number and a unit fraction. **Developing the Big Idea:** In this lesson, students will learn to multiply a whole number by a fraction as repeated addition of a fraction.	**Solve & Share:** Consider giving students an opportunity to use tools and other representations to solve the problem. Share student strategies who may have used concrete tools, representations like a bar diagram, repeated addition or multiplication to solve the problem. As an extension, consider having students compare the two different quantities. **Look Back:** Consider facilitating a discussion around the *Look Back!* as students revisit equivalency. **Visual Learning:** Consider reading the *Prevent Misconceptions* prior to teaching the lesson (TE, p. 550). In the *Visual Learning Animation*, students learn to multiply using repeated addition by drawing pictures or drawing a number line to show the distance Dori walks. Consider having students solve the problem before showing the video. **Convince Me:** Consider using the *Convince Me!* to facilitate a discussion as students need to understand that both addition and multiplication can be used, due to joining equal-sized groups. **Another Example:** Consider facilitating a discussion based on the connection between the *Convince Me!* and *Another Example!* to show how addition and multiplication can be used. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* to reinforce the unit fraction.
Lesson 10-3: Multiply a Fraction By a Whole Number- Use Models		
4.NF.B.4b **4.NF.B.4a** **4.NF.B.4c** MP.2 **MP.4** MP.6 MP.7	**Access Prior Learning**: In the previous lesson, students used models to multiply whole numbers by a fraction. **Beginning of the Big Idea:** In this lesson, students will learn and apply two procedures to multiply fractions by a whole number.	**Solve & Share:** Provide students the opportunity to use tools and/or models to solve the problem. Consider removing the "Think Bubble" from the *Solve & Share* to formally assess student's strategies used. The *Solve & Share* is considered a rate problem as students are given the unit ratio to find how many cups of orange juice for 8 gallons. This may be more difficult to access for students; yet is a necessary part of the standards in 4th grade (multiplicative comparison). **Visual Learning:** Consider reading the *Prevent Misconceptions* before teaching the lesson (TE, p. 556). Be cautious when showing students, a procedure for multiplying fractions. Connect the procedures to the Associative Property and consider letting students make connections between the procedures and models by giving them an opportunity to use what they have previously learned. **Independent Practice/Math Practices and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider giving students multiple opportunities to use various strategies, tools and models to complete the problems. **Assess and Differentiate/Intervention Activity:** Consider encouraging students to use tools or representations before having students apply the procedures learned in the *Visual Learning*. ***CTC: *Solve & Share*** (student work samples) ***-continues on next page-***

		Consider utilizing this question format during practice: **Example Stem 2:** Decide whether each expression is equal to $5 \times \frac{2}{4}$. Click in the table to respond. |	**Equal to** $5 \times \frac{2}{4}$	**Not Equal to** $5 \times \frac{2}{4}$ | |---|---|---| | $2 \times \frac{1}{20}$ | | | | $2 \times \frac{5}{4}$ | | | | $\frac{5 \times 2}{10}$ | | |
Lesson 10-4: Multiply a Whole Number and a Mixed Number		
4.NF.B.4c MP.1 MP.3 MP.7	**Access Prior Learning**: In Topic 3, students learned how to use the distributive property and partial products to multiply whole numbers. In Topic 9, they learned how to rename a mixed number as a fraction greater than one. In the previous lessons, students learned how to multiply a whole number by a fraction. **Beginning of the Big Idea:** In this lesson, students will use previous knowledge to multiply a whole number by a mixed number.	**Possible 2-day Lesson:** Consider expanding this lesson over 2 days to elicit more time for students to work with multiplying mixed numbers using the Distributive Property of Multiplication. Remember by taking two days for this lesson, it will be one of the F/D/E days on the WCSD Pacing Framework. **Day 1:** **Solve & Share:** Consider encouraging students to use tools and representations to solve the problem. **Look Back:** As students are sharing the strategies used in *Solve & Share*, consider facilitating a discussion around the *Look Back!* as students need to understand the context of the problem type (number of groups unknown) and why they multiply. **Visual Learning:** In the *Visual Learning Animation*, students learn how to multiply fractions by using the Distributive Property and partial products. Students also learn how to multiply mixed numbers by changing the mixed number to a fraction greater than one (improper fraction). Consider giving students the opportunity to work out each of these strategies. **Convince Me:** Facilitate a discussion around the *Convince Me!* during the *Visual Learning Animation*, so students make connections as the model is shown. **Day 2:** **Independent Practice/Math Practices and Problem Solving:** Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider giving students multiple opportunities to use various strategies, tools and models to complete the problems. Consider facilitating a discussion around item 19, as it is a rate, multi-step problem. Students may need support in accessing this item. Consider facilitating a discussion around item 20, as students need critique the reasoning of others. **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students, as the activity has students compare multiplying a whole number with multiplying a mixed number. Students use an open array and partial product to show the two problems. Consider facilitating a discussion around how the fractional part can be treated as another place value.
Lesson 10-6: Math Practices and Problem Solving- Model with Math		
4.NF.B.4c **4.NF.B.3d** **4.MD.A.2** **MP.4** MP.1 MP.2	**Access Prior Learning**: In previous topics and lessons, students have modeled and solved problems in multiplication of whole numbers, dividing whole numbers and adding and subtracting fractions and mixed numbers.	**Note**: **Consider using Lesson 10-5 after Lesson 10-6 as 10-5 uses all the operations to solve for time, and 10-6 focus is on multiplication of fractions and mixed numbers.** **Solve and Share:** Consider removing the bar diagrams to elicit more student strategies and increase the cognitive demand. Also, consider having students estimate before solving the problem. ***-continues on next page-***

MP.6	**Developing the Big Idea:** In this lesson, students continue to model and solve problems involving multiplying whole numbers and fractions or mixed numbers.	**Look Back:** Consider using the *Look Back!* as students write number sentences that go along with the *Solve & Share* problem. **Convince Me:** Consider facilitating a discussion around the *Convince Me!* as students make connections between a mixed number and a fraction greater than one (improper fraction). Students also continue developing their understanding of equivalency. **Guided Practice:** Consider encouraging students to use tools and other representations to solve the *Guided Practice* items. ***CTC: *Quick Check*** (digital platform) Consider asking students to draw and shade models utilizing similar phrasing to the following: **Example Stem:** There are 7 people at a picnic. Each person drinks $\frac{2}{3}$ of a liter of lemonade. **Part A:** Each pitcher holds 1 liter. Click on the pitchers to shade the amount of lemonade needed for the picnic. Use the fewest number of pitchers possible. **Part B:** Click the total amount of lemonade that is needed.  **Rubric:** **Part A:** (1 point) The student correctly shades the model to represent the product (e.g., $4\frac{2}{3}$). **Part B:** (1 point) The student selects the correct product (e.g., $\frac{14}{3}$).
Lesson 10-5: Solve Time Problems		
4.MD.A.2 **4.NF.B.4c** MP.1 MP.2 MP.3 **MP.4** MP.5	**Access Prior Learning**: In third grade, students learned relationships between units of time and solved problems by adding and subtracting time in minute intervals. **Beginning of the Big Idea:** In this lesson, students solve problems using all four operations with intervals of time.	Consider giving students opportunity to use tools and representations to answer the different problems in this lesson. Also, focus on equivalency and regrouping. **Assess and Differentiate/Intervention Activity:** Consider doing the *Intervention Activity* with students who may still be struggling with elapsed time.

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Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

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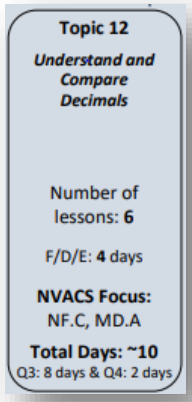
Grade 4 Topic 12: Understand and Compare Decimals



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: Number and Operations- Fractions (pp. 121-125)

Prior to instruction, view the Topic 12 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 623A-623F), the Topic Planner (pp. 623I-623K), all 6 lessons, and the Topic Assessments (pp. 669-670A).

Mathematical Background: Read Cluster Overview- (TE, pp. 623A-623F)	**Topic Essential Questions:** How can you write a fraction as a decimal? How can you locate points on a number line? How can you compare decimals? *Reference TE (p. 623) and Answering the Topic Essential Questions (TE, pp. 667-668) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

12-1	12-2	12-3	12-4	12-5	12-6	Assessment

4 F/D/E days used strategically throughout the topic

Instructional note:

This topic is the last of the cluster group for standard 4.NF.C, "understand decimal notation for fractions, and compare decimal fractions" (Nevada Academic Content Standards (NVACS), 2010). Focus instruction on standards 4.NF.C.5-7, "express a fraction with a denominator 10 as an equivalent fraction with a denominator of 100, use decimal notation for fractions with denominators of 10 and 100, and compare two decimals to hundredths by reasoning about their size" (NVACS, 2010).

"A significant goal of instruction in decimal and fraction numeration should be to help students see that both systems represent the same concepts" (Van de Walle, et al., 2010, p. 329). There are different ways to help students see the connection between fractions and decimals. "Use familiar fraction concepts and models to explore rational numbers that are easily represented by decimals, see how the base-ten system can be extended to include numbers less than 1 as well as large numbers, help children use models to make meaningful transitions between fractions and decimals" (Van de Walle, et al., 2010, p. 329). "Deliberate attention must be given to helping students see that using a decimal is a way of extending the place value system to include numbers less than 1" (Small, 2014, p. 59). One way to see the connection between decimal and fraction numeration is by using tools like place value blocks or hundredths grids. Models are important when students are beginning to understand fractional concepts.

"The role of the decimal point is to designate the units position, and it does so by sitting just to the right of that position" (Van de Walle, et al., 2010, p. 331). "Often students ask why there is no "oneths" place, and one can see why they might ask this. One way to help students is to emphasize that the decimal point is actually in its own column; it is a "marker" that goes with the ones. The line of symmetry goes through the ones column and is not to the right of it" (Small, 2014, p. 59).

Common misconceptions when students are comparing decimals is "0.4 < 0.19 because 4 < 19. The problem is that the units are different: The 4 is 4 tenths, but the 19 is 19 hundredths. Many students will understand this if the analogy is made to measurement units or using visual models" (Small, 2014, p. 61). Consider using base-10 blocks with students who may struggle with these ideas. Use the "flat" representing to represent the "whole" or one, a "long" representing a tenth, and the "unit" representing the hundredth to explore and compare fractions.

Focus Math Practice 7: **Look for and make use of structure**

Focus opportunities for students to develop *Mathematical Practice 7* behaviors, as this is the focus of the Math Practices and Problem Solving. Reference the Teacher's Edition and the NVACS (2010, p. 8).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
tenth hundredth decimal decimal point	

Additional terminology that students may need support with: fractions, numerator, denominator, digit

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students using various strategies to make connections between decimals and fractions?"

Lesson	Evidence	Look for
12-4	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • student strategies and models within a multi-step problem. • student understanding of the relationship between tenths and hundredths.
12-6	***Quick Check*** **(**digital platform)	Focus CTC on the big idea: • students sequencing of decimals on a number line and accurate comparisons of decimals within different place value units. Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 667-670	Use *Scoring Guide* TE pp. 667-670A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 12-1: Fractions and Decimals		
4.NF.C.6 MP.2 MP.3 MP.4	**Access Prior Learning**: In previous topics, students encountered fractions with denominators of 10 and 100. They compared these fractions to other fractions and found equivalent fractions. **Beginning of the Big Idea:** In this lesson, students will learn to write these fractions in decimal form. Students will also relate money to decimals and write money amounts with the dollar sign and decimal point.	**Note:** Consider making an anchor chart with various representations of fractions and decimals. Add to the anchor chart as you progress through the lesson. **Solve & Share:** Consider having tools like place value blocks, counters and representations, like the hundredths grid paper available for students to use. **Look Back:** Consider including the *Look Back!* with the *Solve & Share* problem. **Visual Learning:** Consider reading the *Prevent Misconceptions* before teaching the lesson (TE, p. 628). In the *Visual Learning*, students encounter decimals. Consider having grid paper and place-value blocks available for students to use as the discussion is taking place. **Convince Me:** Consider facilitating a discussion around the *Convince Me!* as students connect equivalency to the problems. Consider having grid paper or place-value blocks for students to use. Verbalize six-tenths and sixty-hundredths, to show they are the same value although they have different place-value positions. **Another Example:** In the *Another Example!*, students represent money using fractions and decimals. Consider having tools, including money or the hundredths grid available for students to use during the discussion. ***-continues on next page-***

		Independent Practice/Math Practices and Problem Solving: Consider facilitating a discussion around item 10 as it re-visits estimation of fractions. **Assess and Differentiate/Intervention Activity:** Consider doing the *Intervention Activity* with all students as students make connections by using a place-value chart to represent decimal values. Consider using place-value charts with students to see where decimals are in relation to whole numbers discussed in Topic 1. Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** Enter a decimal that is equivalent to $\frac{3}{10}$.
Lesson 12-2: Fractions and Decimals on the Number Line		
4.NF.C.6 MP.1 MP.2 MP.4 MP.6 MP.7	**Access Prior Learning**: In the previous topic, students represented equivalent fractions on a number line. **Beginning of the Big Idea:** In this lesson, students will locate given decimals on a number line and name the decimal at a given point.	**Solve & Share:** Consider giving students an opportunity to use tools and other representations to write decimal numbers. Students need to pay attention to the whole and the number of parts. **Look Back:** Consider facilitating a discussion around the *Look Back!*, as students connect the new number line to the number lines in the *Solve & Share*. Consider emphasizing that fractions are equal parts of the whole. **Visual Learning:** Consider reading the *Prevent Misconceptions* prior to teaching the lesson (TE, p. 634). In the *Visual Learning*, students learn to use two different strategies to find decimals on a number line. **Note:** Consider facilitating a discussion around Part C in the *Visual Learning Animation* as students learn that there are infinite rational numbers on a number line. **Convince Me:** Consider using the *Convince Me!* to facilitate a discussion as students determine which decimal is not placed correctly on a number line and explain. **Independent Practice/Math Practices and Problem Solving:** Consider facilitating a discussion around the "Higher Order Thinking" item 24, whole group, as students can see there are many numbers on a number line. Many problems in the *Independent Practice* have students work with numbers greater than one. Consider choosing items 7-12 for students to complete.
Lesson 12-3: Compare Decimals		
4.NF.C.7 **4.MD.A.2** MP.2 MP.3 MP.5	**Access Prior Learning**: In the previous topics, students learned that the digit in one place of a whole number represents 10 times as much as it represents in the place to its right. **Developing the Big Idea:** In this lesson, students compare decimals by using place value understanding and models.	**Look Back:** Consider facilitating a discussion around the *Look Back!* as students think about the comparison of decimals referring to the same whole. Students will make connections between fractions and decimals. **Visual Learning:** Consider reading the *Prevent Misconceptions* before teaching the lesson (TE, p. 640). **Another Example:** Consider facilitating a discussion around the *Another Example!* as students are using new representations of place-value blocks. For example, the flat is now 1 instead 100. **Assess and Differentiate/Intervention Activity:** Consider having all students do the *Intervention Activity* as students will be comparing 0.09 to 0.99 by using a place-value chart, other representations or tools. Consider extending this activity by asking students, "How would it be different if you were to compare 0.9 to 0.99?" Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** Select the symbol (< , > , or =) that correctly compares each pair of numbers. | | < | > | = | | 0.09 ☐ 0.7 | | | | | 1.2 ☐ 0.37 | | | |

Lesson 12-4: Add Fractions with Denominators of 10 and 100

4.NF.C.5

MP.1
MP.3
MP.4
MP.5

Access Prior Learning:
In the previous topics, students learned how to find equivalent fractions by multiplying and using models. Students also learned how to add and subtract fractions with like denominators.

Beginning of the Big Idea:
In this lesson, students add fractions with denominators of 10 and 100, renaming fractions to have common denominators.

Solve & Share:
Consider removing the grid, but have grid paper and place-value blocks available for students to use.

Look Back:
Consider facilitating a discussion around the *Look Back!* as students think about how much remains after the *Solve & Share* discussion. Consider giving students the opportunity to solve with fractions and then convert the answer to a decimal.

Visual Learning:
Consider having students use grid paper or place-value blocks to solve the problem before introducing the procedure.

Independent Practice/Math Practices and Problem Solving:
Students do not need to do all the problems in their Student Edition. Ask students to complete the *Quick Check* items (marked with a pink check mark) first and continue on to other items as appropriate. Consider providing students multiple opportunities to use various strategies, tools and models to complete the problems.
Item 25 is a rate problem. Consider facilitating a discussion around this item as students may find difficulty in answering the question.

Consider utilizing this question format during practice:

Example 1
Full Statement

Example Stem: Determine if each equation is true or false. Select True or False for each equation.

	True	False
$\frac{5}{10} + \frac{18}{100} = \frac{68}{100}$		
$\frac{11}{10} + \frac{13}{100} = \frac{24}{100}$		
$\frac{10}{10} + \frac{45}{100} = \frac{145}{100}$		

Consider modifying questions to include the following phrases:

Example 1
Full Statement

Example Stem 1: Enter the unknown numerator that makes this equation true.

$\frac{6}{10} + \frac{3}{100} = \frac{\square}{100}$

Example 1
Full Statement

Example Stem 1: Enter the unknown numerator that makes this equation true.

$\frac{\square}{10} + \frac{15}{100} = \frac{65}{100}$

Example 2
Full Statement

Example Stem 2: Enter the unknown number that makes this equation true.

$\frac{3}{10} + \square = \frac{65}{100}$

CTC: *Solve & Share (student work samples)

Lesson 12-5: Solve Word Problems Involving Money

4.MD.A.2
4.NF.C.6

MP.1
MP.2
MP.4
MP.7

Access Prior Learning:
In second grade, students used coins and bills to represent money amounts. In previous topics, students developed fluency for addition and subtraction with whole numbers. In 12-1, students wrote money amounts using dollar signs and decimal points.

Solve & Share:
Consider modifying the problem by changing $24 to $24.50. This will give students the opportunity to work with decimals and change.

Look Back:
Consider having students complete the *Look Back!* while they work on the *Solve & Share* as the *Look Back!* reinforces estimation.

-continues on next page-

MP.8	**Developing the Big Idea:** In this lesson, students use bills and coins to represent money amounts and to do computations.	**Another Example:** In the *Another Example!*, students use models to divide dollar amounts. Consider giving students opportunity to use concrete tools to solve the problem. Refrain from showing students the long division algorithm. Students will have an opportunity to learn long division in future grades. **Independent/Math Practices and Problem Solving:** Consider having students work on items 7 and 9 as students estimate.
Lesson 12-6: Math Practices and Problem Solving- Look For and Use Structure		
4.NF.C.7 **4.MD.A.2** **MP.7** MP.1 MP.2 MP.3 MP.4 MP.6	**Access Prior Learning**: In previous topics, students have used MP7: Look for and Use Structure. **Developing the Big Idea:** In this lesson, students will focus on thinking habits good problem solvers use when they look for and use structures.	**Visual Learning:** The *Visual Learning* reinforces some common fraction-decimal equivalence. For example, instead of using ½ the animation uses 0.5. Consider facilitating a discussion around the relationships between fractions and decimals. **Assess and Differentiate/Intervention Activity:** Consider facilitating a discussion with all students, as students see the equivalence between fractions and decimals on a number line. **Independent Practice/Math Practices and Problem Solving:** Consider facilitating a discussion around items 6-11, as students use patterns to solve money problems. Students will see more pattern work in Topic 14.

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Small, M. (2014). *Uncomplicating fractions to meet common core standards in math, K-7.* New York, NY: Teachers College Press, Nelson Education.

Van de Walle, J.A., Karp, K., Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally*. New York, NY: Pearson.

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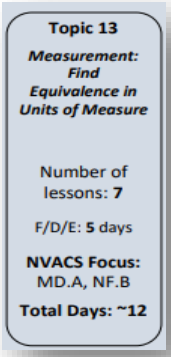
Grade 4 Topic 13: Measurement: Find Equivalence in Units of Measure

Big Conceptual Idea: Measurement and Data (Measurement Part) (pp. 87-92)
Prior to instruction, view the Topic 13 Professional Development Animation located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 671A-671F), the Topic Planner (pp. 671I-671J), all 7 lessons, and the Topic Assessments (pp. 727-728A).



Mathematical Background:	**Topic Essential Questions:**
Read Cluster Overview- (TE, pp. 671A-671F)	How can you convert from one unit to another? How can you be precise when solving math problems? *Reference TE (p. 671) and Answering the Topic Essential Questions (TE, pp. 725-726) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

13-1	13-2	13-3	13-4	13-5	13-6	13-7	Assessment

5 F/D/E days used strategically throughout the topic.

Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Instructional note:
This topic focuses on converting units of measure and explores area and perimeter concepts. Focus for standard 4.MD.A, "solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit" (Nevada Academic Content Standards (NVACS), 2010). 4.MD.A.1 and 4.MD.A.3 focus on relative size of measurements, find equivalence among measurements of the same attribute and apply the area and perimeter formula for rectangles in real world and mathematical problems.

"Technically, a *measurement* is a number that indicates a comparison between the attributes of the object (or situation, or event) being measured and the same attribute of a given unit of measure. To measure means that the attribute being measured is "filled", "covered" or "matched". (Van de Walle, Karp, Bay-Williams, 2010, p. 370).

For measurement, "research indicates that when students see standard rulers with the numbers on the hash marks, they often believe that the numbers are counting the marks rather than indicating the units or spaces between the marks" (Van de Walle, et al., 2010, p. 376). Consider making a connection between the spaces on a ruler to the spaces on a number line. Two assessments teachers might use with students to check for understanding when measuring, is to give students a ruler with hash marks and no numbers or give students a 'broken ruler" with the first two units broken off (Van de Walle, et al., 2010, p. 376). In using these assessments, teachers can gain in-sight into students' knowledge of measuring before having students convert measurements.

In this topic, consider giving students multiple opportunities to use concrete tools or representations to support them as they work with abstract concepts.

SBAC connection: Please note that students must develop an understanding of measurement conversions and be able to access this knowledge without outside support. Thus, students must know from memory all conversions within the units outlined in the standard.

Focus Math Practice 6: Attend to precision
Focus opportunities for students to develop *Mathematical Practice 6* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 12-7. Reference the Teacher's Edition (pp. F26-F26A) and the NVACS (2010, p. 7).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary
Use these words consistently during instruction.

New Academic Vocabulary: (First time explicitly taught)		**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)	
quart	pound	*capacity*	*milligram*
gallon	ton	*weight*	*kilogram*
cup	millimeter	*centimeter*	*perimeter*
pint	meter	*milliliter*	*area*
fluid ounce	kilometer	*liter*	*formula*
ounce	mass	*gram*	

Additional terminology that students may need support with: convert, units, customary units and metric units

*Consider adding measurement terminology to an anchor chart as they are discussed within the lessons.

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding questions: "What strategies are students using to convert measurements?"
"Are students able to determine measurements based on given information?"

Lesson	Evidence	Look for
13-3	***Quick Check*** (digital platform) Item 2	Focus CTC on the big idea: • students use the information on a line plot to convert pounds to ounces. Printable version available under "Teacher Resources".
13-6	***Math Practice and Problem Solving*** (student work samples) Item 15	Focus CTC on the big idea: • students determine the dimensions of the rectangle based on the given area.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 725-728	Use *Scoring Guide* TE pp. 725-728A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Math Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 13-1: Equivalence with Customary Units of Length		
4.MD.A.1 **4.MD.A.2** **4.NF.B.3d** **4.NF.B.4c** MP.1 **MP.6** MP.7 MP.8	**Access Prior Learning**: In previous grades, students estimated and measured length to the nearest inch or foot. Students measured the same object with two different units to gain an understanding of the principle that the smaller the unit, the greater the number of units needed. **Beginning of the Big Idea:** In this lesson, students use the relative size of customary units of length to convert from a larger unit to a smaller unit.	**Note:** Consider teaching this lesson over 2 days. Remember, this will count towards one of the F/D/E days in the WCSD Pacing Framework. **Day 1:** Consider giving students the opportunity to measure items using the customary unit of measure. Have students estimate and then measure with a ruler. Consider having students write what they measured, the estimation and the actual measurement amount on a sticky note (have students measure a few items or objects). Next, have students go to a sticky note and measure to critique the reasoning of others. If they disagree with a student, they need to justify why they disagree. (Idea from Mary Wilson, Pearson representative). Child-watch for students who are struggling with reading a ruler. Also look for students who may already be converting inches to feet, feet to yards and vice versa. Note: Students have been working with measuring using tools in both U.S. Customary and the Universal System of Measures (Metric System) since 2nd Grade. **Day 2:** **Note:** When students are converting between measurements, they need to understand the two measurements are equivalent. **Solve & Share:** Consider giving students an opportunity to solve the *Solve & Share* using yard sticks or other tools to see how many feet are in 75 yards. Consider removing the diagram as it may not be helpful for students when answering the question. **Look Back:** Consider connecting the *Look Back!* to the *Solve & Share* as students think about a multiplicative comparison between yards and feet. **Visual Learning:** Consider reading the *Prevent Misconceptions* and questions before teaching the lesson (TE, p. 680). Consider giving students an opportunity to solve the *Visual Learning* on their own and then facilitate a discussion around the problem as it is a multi-step problem. ***-continues on next page-***

		Convince Me: Consider connecting the *Convince Me!* back to Day 1 when students measured various objects. Have students think about when they measured in inches and feet. Did they notice any structures when measuring in a smaller unit versus a larger unit? **Another Example:** Compare the *Another Example!* to the *Visual Learning*. In the *Visual Learning Animation*, students convert from feet to inches and in the *Another Example!*, students convert yards to inches. Have students reflect on patterns they notice when converting. **Assess and Differentiate/Intervention Activity:** Consider facilitating a discussion around the *Intervention Activity* with all students. Guide students in a discussion about relative size. Consider utilizing this question format during practice: **Example 1** Full Statement **Example Stem:** Decide if each measurement is equal to 5 yards. Select Yes or No for each measurement. | | Yes | No | | 180 inches | | | | 27 inches | | | | 15 feet | | |
Lesson 13-2: Equivalence with Customary Units of Capacity		
4.MD.A.1 **4.MD.A.2** **4.NF.B.3d** **4.NF.B.4c** MP.1 MP.2 MP.8	**Access Prior Learning**: In the previous lesson, students learned how to convert a larger customary unit of length to a smaller unit. **Beginning of the Big Idea:** In this lesson, students will learn the relative size of customary units of capacity to convert from a larger unit to a smaller unit.	**Solve & Share:** Consider providing students with an opportunity to use tools to solve the problem. Students are given a table with conversion amounts, so consider giving them an opportunity to figure out the table on their own and solve the problem before explaining the table to them. **Visual Learning:** Students use a recipe to convert different customary units of capacity. Consider having students bring in their own recipe to use for the *Visual Learning*. Relate the *Visual Learning* to fraction strips instead of focusing attention on "Gallon Man". **Independent Practice/Math Practices and Problem Solving:** Consider facilitating a discussion around item 15 with students. Students are asked to use the information from a table to solve a problem. **Assess and Differentiate/Intervention Activity:** Consider facilitating a discussion around the *Intervention Activity* as students use "realia" to find equivalent capacities.
Lesson 13-3: Equivalence with Customary Units of Weight		
4.MD.A.1 **4.MD.A.2** **4.NF.B.3d** **4.NF.B.4c** MP.1 MP.2 **MP.6** MP.8	**Access Prior Learning**: In the previous lessons, students converted between a larger unit and a smaller unit using customary units of measure and capacity. **Beginning of the Big Idea:** In this lesson, students learn about customary units of weight; the relative size of the units and how to convert from a larger weight to a smaller weight.	**Visual Learning:** Consider reading the *Prevent Misconceptions* prior to teaching the lesson (TE, p. 692). The *Visual Learning* is a multi-step problem. Consider giving students the opportunity to solve the problem before facilitating a discussion. **Independent Practice/Math Practices and Problem Solving:** Consider facilitating a discussion around items 13 and 14 as students think about patterns. This will support the work in Topic 14. ***CTC: *Quick Check item 2* (digital platform)
Lesson 13-4: Equivalence with Metric Units of Length		
4.MD.A.1 **4.MD.A.2** MP.1 MP.3	**Access Prior Learning**: In previous grades, students estimated and measured length to the nearest centimeter or meter. In 13-1, students converted customary units of length.	**Note:** Consider connecting the metric system to the base-ten system. **Solve & Share:** Consider having students solve the problem without a discussion about the ruler's marks. This will give you an opportunity to child-watch to see how students measure in centimeters and millimeters, and if they know the difference between the two measurements of length. ***-continues on next page-***

<table>
<tr><td>MP.5
MP.6
MP.8</td><td>Developing the Big Idea:
In this lesson, students will learn the relative size of metric units of length by converting from a larger unit of length to a smaller unit.</td><td>Look Back:
Consider facilitating a discussion around the Look Back!. Consider giving students an opportunity to use a ruler as a tool to support students in finding the answer.

Visual Learning:
Consider reading the Prevent Misconceptions prior to teaching the lesson (TE, p. 698).

Convince Me:
Consider using the Convince Me! as a formative assessment or facilitate a discussion around equivalence.

Independent Practice/Math Practices and Problem Solving:
Consider facilitating a discussion around items 10 and 11, as students make connections through patterns. See if students use our base-ten system as they work on these items.</td></tr>
<tr><td colspan="3">Lesson 13-5: Equivalence with Metric Units of Capacity and Mass</td></tr>
<tr><td>4.MD.A.1
4.MD.A.2

MP.1
MP.3
MP.5
MP.6
MP.8</td><td>Access Prior Learning:
In third grade, students estimated and measured capacity and mass using metric units.

Beginning of the Big Idea:
In this lesson, students will learn the relative size of metric units of mass and capacity. Students will also convert from a larger metric unit of capacity or mass to a smaller unit.</td><td>Solve & Share:
Consider having students use their own water bottle to help solve the problem.

Look Back:
This may be an opportunity to relate the units to the idea of looking at the same whole when converting.

Visual Learning:
In the Visual Learning Animation, students solve problems related to capacity. Consider facilitating a discussion around capacity.

Another Example:
In the Another Example!, students solve problems related to mass. Consider facilitating a discussion around mass.</td></tr>
<tr><td colspan="3">Lesson 13-6: Solve Perimeter and Area Problems</td></tr>
<tr><td>4.MD.A.3
4.MD.A.2
4.NBT.B.5
4.NF.B.4c

MP.1
MP.2
MP.3</td><td>Access Prior Learning:
In third grade, students learned the meaning of area as the number of square units needed to cover a plane. In Topic 4, students used an area model to multiply whole numbers.

Developing the Big Idea:
In this lesson, students solve problems using perimeter and area.</td><td>Solve & Share:
Consider removing the picture of the wall given to students. Consider giving students the opportunity to use tools or representations, like grid paper to solve the problem.

Visual Learning:
Read the Prevent Misconceptions prior to the lesson to clarify students' misconceptions regarding area and perimeter (TE, p. 710). Consider giving students the opportunity to solve for perimeter and area before showing the animation. See if students can explain how they solved the problems before giving students the perimeter and area formulas.

Convince Me:
Consider facilitating a discussion around the Convince Me!. Students are given the area of the park, and solve for the dimensions and the perimeter of the park.

Assess & Differentiate/Intervention Activity:
Consider using the Intervention Activity with students who may be struggling with area and perimeter as students find the area and perimeter of rectangles using grid paper.

Consider utilizing the following question formats during practice:

Example 1
Full Statement

Example Stem 1: The dimensions for three rectangles are shown. Decide if each rectangle has an area equal to 100 square feet. Select Yes or No for each rectangle.

<table><tr><th></th><th>Yes</th><th>No</th></tr><tr><td>Rectangle 1:
• Length = 20 ft
• Width = 5 ft</td><td></td><td></td></tr><tr><td>Rectangle 2:
• Length =10 ft
• Width = 10 ft</td><td></td><td></td></tr><tr><td>Rectangle 3:
• Length = 25 ft
• Width = 4 ft</td><td></td><td></td></tr></table>

-continues on next page-</td></tr>
</table>

<table>
<tr>
<td></td>
<td></td>
<td>
Example 2
Full Statement
Example Stem 2: The dimensions for three rectangular gardens are shown. Decide if each garden has a perimeter equal to 100 meters. Select Yes or No for each garden.
<table>
<tr><th></th><th>Yes</th><th>No</th></tr>
<tr><td>Garden 1:
• Length = 45 m
• Width = 5 m</td><td></td><td></td></tr>
<tr><td>Garden 2:
• Length = 50 m
• Width = 50 m</td><td></td><td></td></tr>
<tr><td>Garden 3:
• Length = 25 m
• Width = 4 m</td><td></td><td></td></tr>
</table>
*CTC: Math Practice & Problem Solving item 15 (student work samples)
</td>
</tr>
<tr><td colspan="3">Lesson 13-7: Math Practices and Problem Solving- Precision</td></tr>
<tr>
<td>4.MD.A.2
4.MD.A.3
4.NBT.B.5
4.NF.B.4

MP.6
MP.1
MP.2
MP.4</td>
<td>Access Prior Learning:
In previous topics, students have had an opportunity to work with Math Practice 6.

Developing the Big Idea:
In this lesson, students will focus on the thinking habits that good problem solvers use, such as precision to solve measurement problems.</td>
<td>Solve & Share:
Consider providing students the opportunity to use tools or representations to solve the problem. As an extension, consider relating this problem back to fractions by asking students, “What fraction of the wall does one poster cover?” This extension may support students as they work on the Visual Learning.

Look Back:
Consider using the Look Back! as an extension to the Solve & Share.

Convince Me:
In the Convince Me! students are asked to use mathematical language to explain how to solve the Visual Learning. Consider facilitating a discussion on how to use mathematical language to make an explanation clear.</td>
</tr>
</table>

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). Measurement and data, measurement.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

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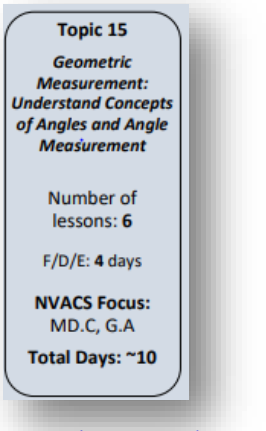
▸ Grade 4 Topic 15: Geometric Measurements: Understand Concepts of Angles and Angle Measurements



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: Measurement and Data (Measurement Part) (pp. 87-92)
Prior to instruction, view the Topic 15 Professional Development Video (located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 765A-765F), the Topic Planner (pp. 765I-765J), all 6 lessons, and the Topic Assessments (pp. 813-814A).

Mathematical Background:	**Topic Essential Questions:**
Read Cluster Overview- (TE, pp. 765A-765F)	What are some common geometric terms? How can you measure angles? *Reference TE, p. 765 and Answering the Topic Essential Questions (TE, pp. 811-812) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

15-1	15-2	15-3	15-4	15-5	15-6	Assessment

4 F/D/E days used strategically throughout the topic.

Instructional note:
This topic focuses on angles and angle measurements. Focus for standard 4.MD.C, "Geometric measurement: understand concepts of angle and angle measures" (Nevada Academic Content Standards (NVACS), 2010). 4.MD.C.5a-b, 4.MD.C.6 and 4.MD.C.7 focus on recognizing angles as geometric shapes that are formed wherever two rays share a common endpoint, understand concepts of angle measurement, measure angles in whole-number degrees using a protractor, and recognize angle measures as additive.

"Technically, a *measurement* is a number that indicates a comparison between the attributes of the object (or situation, or event) being measured and the same attribute of a given unit of measure. To measure means that the attribute being measured is "filled", "covered", or "matched". (Van de Walle, Karp and Bay-Williams, 2010, pg. 370). "The attribute of angle size might be called the "spread of the angle's rays." Angles are composed of two rays that are infinite in length with a common vertex. The only difference in their size is how widely or narrowly the two rays are spread apart" (Van de Walle, et al., 2010, pg. 386).

John Van de Walle, et al., (2014) states,

> The two tools commonly used for measuring angles are angle rulers and protractors. The protractor is one of the most poorly understood measuring instruments. Part of the difficulty arises because the units (degrees) are so small. It would be physically impossible for students to cut out and use a single degree to measure an angle accurately. In addition, the numbers on most protractors run clockwise and counterclockwise along the edge, making the scale hard to interpret without a strong conceptual foundation. Note that the units of degrees are based on an angle in which the vertex of the rays is located at the midpoint of a circle creating an arc. A "one degree" angle is one in which the arc is 1/360 of the circle (pp. 336-337).

Give students multiple experiences with angle measures before giving students a protractor, so they understand the reason for the half circle the protractor represents. Using a full circle protractor can help students with the angle measurements conceptually, as well as having them make one themselves.

Focus Math Practice 5: Use appropriate tools
Focus opportunities for students to develop *Mathematical Practice 5* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 15-6. Reference the Teacher's Edition (pp. F25-F25A) and the NVACS (2010, p. 7).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
point line segment ray acute angle obtuse angle straight angle reflex angle degree unit angle angle measure protractor	*line* *right angle* *vertex*

Additional terminology that students may need support with: measure, circle

*Collaborative Team Conversations (CTC)

Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

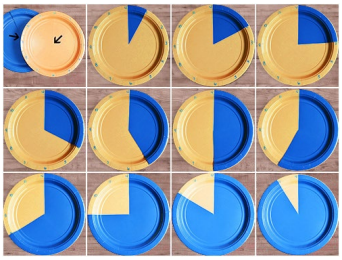
Guiding questions: "What tools or strategies are students using to determine the measurement of angles?"
"Are students able to find angle measurements based on the information provided?"

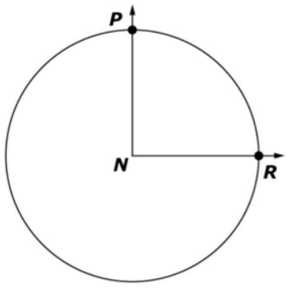
Lesson	Evidence	Look for
15-2	***Math Practice and Problem Solving*** (student work samples) Item 13	Focus CTC on the big idea: • students use their understandings of angle measurements of a circle to find the unit angle.
15-4	***Quick Check*** (digital platform) Items 1, 2 and 5	Focus CTC on the big idea: • students find the angle measurements based on the given information. • students use a protractor to find the angle measurement. Printable version available under "Teacher Resources".

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 811-814	Use *Scoring Guide* TE pp. 811-814A

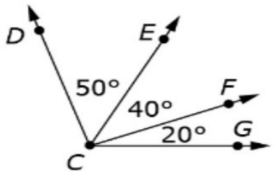
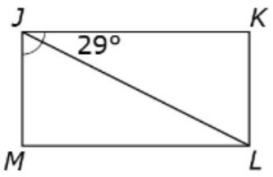
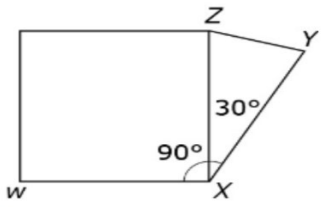
Standards listed in **bold** indicate a focus of the lesson.

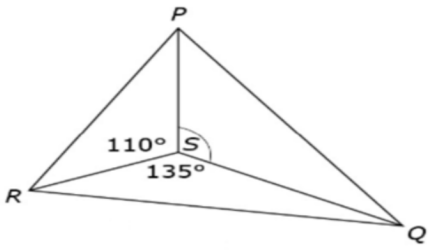
NVACS (Content and Math Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 15-1: Lines, Rays, and Angles		
4.MD.C.5 **4.G.A.1** MP.2 MP.4 MP.6 MP.7	**Access Prior Learning**: In the previous grades, students learned to draw shapes with a given number of angles or sides. Students analyzed shapes by their attributes such as sides and angles. **Beginning of the Big Idea:** In this lesson, students recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angles.	**Note:** Consider changing the orientation of the angles, so students do not develop a misconception that angles only open in one direction. **Look Back:** Consider having students complete the *Look Back!* as they work on the *Solve & Share*. Students will be drawing 3 angles: two less than 90 degrees and one greater than 90 degrees. **Visual Learning:** The *Visual Learning Animation* is heavy with mathematical language related to angles. Consider pausing the video after each new mathematical word and have students "act out" the words. The *Intervention Activity* has students using their bodies to represent the terms, so use the *Intervention Activity* during the animation. Connect what students did in the *Solve & Share/Look Back!* to the *Visual Learning Animation*. **Convince Me:** Have students do the *Convince Me!* in a math journal as students complete the figures to show the given angle. ***-continues on next page-***

		Independent Practice/Math Practices and Problem Solving: As students name line segments, rays and angles, consider making sure students are labeling the different line segments, rays and angles correctly. Consider facilitating a discussion around items 12-14, as students are looking for angles in a diagram and not just in isolation. Consider facilitating a discussion around the "Higher Order Thinking" item 19 with whole class, as students are problem solving and constructing an argument based on Nina's response. **Assess and Differentiate/Intervention Activity:** Consider using the *Center Games* at the bottom of page 775A as students identify and write descriptions about the types of line, rays and angles represented in the diagrams.
Lesson 15-2: Understand Angles and Unit Angles		
4.MD.C.5a MP.1 MP.2 MP.3 MP.4	**Access Prior Learning**: In previous topics, students developed an understanding of the measurement process as a comparison of a unit to a whole. **Beginning of the Big Idea:** In this lesson, students learn that the unit used to measure angles is a degree and that a unit angle is an angle that turns through 1/360 of a circle.	**Note:** Consider breaking this lesson into 2 days. Remember by breaking the lesson into 2 days, this uses one of the F/D/E days allocated in the WCSD Pacing Framework. **Day 1:** **Solve & Share:** Consider changing the *Solve & Share* to the *Homework & Practice* item 7, "Janie served 4 same-size pizzas at the class party. Explain how to find how many slices of the pizza Janie served if the angle for each slice turns through a right angle". Consider having tools and representations available for students to use. After a discussion has been facilitated around the *Homework and Practice* item 7, consider having students complete the *Solve & Share*, students connect what they learned in the previous lesson and Topic 13 to be able to solve the problem. Consider having tools or representations available for students to use. As an extension for all students, ask, "What other angle is made by the two hands on the clock?" **The purpose of this question is to get students to start thinking about the concept of *reflex angles*. See diagram at the end of this Curriculum Guide.** **Look Back:** Consider facilitating a discussion around the *Look Back!* as students connect the *Solve & Share* to fractions. Students will be using this idea to think about a circle and degrees related to the fractions ¼ and ¾. Consider facilitating a discussion around the ***reflex angle*** by connecting ¾ to the degree amount on a circle. The reflex angle is 270 degrees. This idea will help support work in Day 2 when you begin the *Visual Learning Animation*. **Day 2:** **Visual Learning:** Consider reading the *Prevent Misconceptions* prior to teaching the lesson (TE, p. 778). In the *Visual Learning Animation*, students begin to think about angle measurements in degrees and the fractional turns through parts of the circle. Students think about the unit angle 1/360 degrees. Consider having students make a double paper plate circle. See the diagram below. Find the idea for this tool at https://www.mathlearningcenter.org/blog/interactive-paper-plate-fractions.  **Convince Me:** Consider facilitating a discussion around the *Convince Me!* as students compare their understanding of fractions and the same whole to circles and angle measurements. The size of the circle does not change the measure of the angle. ***-continues on next page-***

		Another Example: Consider facilitating a discussion around the *Another Example!* as students think about 45 degrees and the fraction of a circle the angle turns through. **Independent Practice/Math Practices and Problem Solving:** Consider having tools and other representations available for students to use as they work on the problems in the *Independent Practice/Math Practices and Problem Solving*. Tools may include clocks, and the circle made in the *Visual Learning*. **Note:** Students are not using protractors at this time. **Assess and Differentiate/Intervention Activity:** Consider having all students complete the *Intervention Activity* as students think about 6:00 on the clock (1/2) and think about the angle measure that is formed (180 degrees). Consider putting this **before** the *Another Example!*. Consider utilizing this question format during practice or replacing item 4 in the *Guided Practice:* Examples Example 1 Full Statement **Example Stem:** The vertex of $\angle PNR$ is at the center of the circle. The circular arc between Point P and Point R is $\frac{1}{4}$ of the circle.  Enter the measure, in degrees, of ∠PNR in the response box. ***CTC:** ***Math Practices and Problem Solving item 13*** (student work samples)
Lesson 15-3: Measure with Unit Angles		
4.MD.C.5b **4.MD.C.5a** MP.1 MP.3 MP.4 **MP.5** MP.8	**Access Prior Learning**: In the previous lesson, students learned that the unit used to measure angles is a degree, and a unit angle is an angle that turns through 1/360 of a circle. **Developing the Big Idea:** In this lesson, students begin applying the measurement process to measuring angles.	**Solve & Share:** Instead of giving the angle measures, consider having students figure out the smaller angle measure of a tan pattern block before solving the problem. The *Solve & Share* problem gives students the measurement of the angle degree, but consider having students figure out how many smaller angles on the tan pattern block fit in the 90-degree angle or edge of a paper. Then have students use the information found to solve the problem. Child-watch for students who use the rhombi and reasoning skills to show how three rhombi make 90 degrees. Students may determine one rhombus' angle is 30 degrees, because 90 divided by 3 is 30 degrees. Have students share their models, strategies or equations with the class. **Look Back:** Consider changing the *Look Back!* to, "How many right angles make a straight angle? How many 45-degree angles form a straight angle? Explain. **Visual Learning:** Consider providing students an opportunity to use pattern blocks as the discussion is facilitated around the *Visual Learning Animation*. **Convince Me:** Connect the *Convince Me!* to the *Visual Learning Animation* and the previous lesson as students think about one-degree angles in an angle measure. **Guided Practice:** Consider facilitating a discussion around the *Guided Practice* items 3 and 4 as students find angle measurements in squares.

Lesson 15-4: Measure and Draw Angles		
4.MD.C.6 MP.1 MP.2 **MP.5** MP.6	**Access Prior Learning**: In the previous lesson, students used pattern blocks' angles to measure other angles. **Developing the Big Idea:** In this lesson, students are introduced to a tool designed for measuring angles, a protractor.	**Solve & Share:** Students read a protractor to find the measure of an angle. Have students think about if the angle is less than or greater than 90-degrees before figuring out the angle measurement. By estimating the angle, students may read the protractor correctly. Child-watch for students who may struggle when reading a protractor and do not connect the estimation to the angle measurement. **Look Back:** Consider facilitating a discussion around the *Look Back!* as students draw an angle measure of 110-degrees. Have students compare the angle from the *Solve & Share* to the angle in the *Look Back!*. Consider having students estimate how many 110-degree angles are in a circle. **Visual Learning:** Students learn how to use the protractor in the *Visual Learning Animation*. Consider giving students an opportunity to use the tool to make angles, including making a 45-degree angle and a 130-degree angle from the animation. **Convince Me:** Consider facilitating a discussion around the *Convince Me!* or using it as an opportunity for a formative assessment, as it helps clarify any misconceptions students may have reading a protractor. **Independent Practice/Math Practices and Problem Solving:** Consider creating the *Independent Practice* on a separate piece of paper for students to measure, as to change the orientation of the angles and the size of the angles to make it easier for students to measure. For example, item 11 change the 90-degree angle opening to face the opposite direction. This will help students read the protractor and not develop a misconception that angles only face one direction. **Assess and Differentiate/Intervention Activity:** Consider facilitating a discussion around the *Intervention Activity*, as this will support students in the next lesson. ***CTC: *Quick Check items 1, 2 and 5*** (digital platform)
Lesson 15-5: Add and Subtract Angle Measures		
4.MD.C.7 MP.1 MP.2 MP.3 MP.4 MP.7	**Access Prior Learning**: In third grade, students learned that area is additive and solved problems by adding areas. **Developing the Big Idea:** In this lesson, students learn angle measurements are additive. They add and subtract angle measurements to solve problems.	**Solve & Share:** Consider sharing the different ways students divided the angle into two angles. **Look Back:** Facilitate a discussion around the *Look Back!* and make connections to the *Solve & Share*. Have students write the equation they would use for the two angles used to make the angle of 70-degrees. Also, consider facilitating a discussion around the idea that no matter what students divided the angles into, they should still have the same total. **Visual Learning:** In the *Visual Learning Animation*, this is the first time students need to find the missing part of the total angle. Consider having students solve the problem after Part A, and then discuss student strategies before showing the other parts of the animation. **Convince Me:** Consider facilitating a discussion around the *Convince Me!* or use it as a formative assessment. Students find one unknown of three angles for the first time. **Assess and Differentiate/Intervention Activity:** Consider doing the *Intervention Activity* with all students, as students work by making their own angles. ***-continues on next page-***

		Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** Use the diagram to solve the problem. • The measure of $\angle DCE = 50°$. • The measure of $\angle ECF = 40°$. • The measure of $\angle FCG = 20°$.  Enter the measure, in degrees, of $\angle DCG$.
Lesson 15-6: Math Practices and Problem Solving- Use Appropriate Tools		
4.MD.C.6 **4.MD.C.7** **MP.5** MP.1 MP.2 MP.4	**Access Prior Learning**: In previous topics and lessons, students use appropriate tools strategically. **Developing the Big Idea:** In this lesson, students will focus on thinking habits good problem solvers use when they use appropriate tools strategically to solve problems involving angle measures and distances.	**Solve & Share:** Consider having various tools available for students to use to solve the problem. Give them opportunity to use various tools and to problem solve which tools would be best in helping them solve the problem. **Visual Learning:** In the *Visual Learning*, students learn to use a protractor and a ruler in different ways. Consider providing students an opportunity to measure the red trapezoid pattern block and then discuss students' findings before moving on in the animation. **Convince Me:** Consider facilitating a discussion around the *Convince Me!*, as students work on the *Visual Learning* problem. **Guided Practice:** In the *Guided Practice*, students are not asked to find angles, but how to use appropriate tools when adding mixed fractions. **Assess and Differentiate/Intervention Activity:** Consider having all students complete the *Intervention Activity* as students use appropriate tools to find angle measurements. Consider utilizing the following question formats during practice: Example 2 Full Statement **Example Stem 2:** In the figure shown, $JKLM$ is a rectangle and $\angle KJL = 29°$.  Enter the measure, in degrees, of $\angle MJL$. Example 3 Full Statement **Example Stem 3:** A student made the design shown with shapes. • The measure of $\angle WXZ = 90°$. • The measure of $\angle YXZ = 30°$.  Enter the measure, in degrees, of $\angle WXY$. ***-continues on next page-***

		Example 4 Full Statement **Example Stem 4:** A student made the design shown with shapes. • The measure of $\angle PSR = 110°$. • The measure of $\angle RSQ = 135°$.  Enter the measure, in degrees, of ∠PSQ.

An angle

name	measurement
right angle	90°
straight angle	180°
acute angle	between 0 and 90°
obtuse angle	between 90° and 180°
reflex angle	between 180° and 360°

Common Core Standards Writing Team. Measurement and data progression.

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). Measurement and data, measurement.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

Van de Wall, J., Karp, K., & Bay-Williams, J. (2010). *Elementary and middle school mathematics: Teaching developmentally.* Boston, MA: Pearson.

Van de Walle, J., Karp, K., Lovin, L., & Bay-Williams, J. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades 3-5.* (2nd ed.). New York, NY: Pearson.

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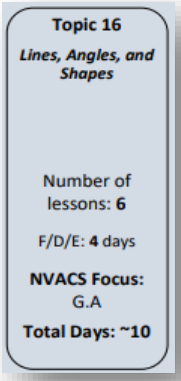
Grade 4 Topic 16: Lines, Angles and Shapes



Big Conceptual Idea: Geometry (pp. 110-111)
Prior to instruction, view the Topic 16 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 815A-815F), the Topic Planner (pp. 815I-815J), all 6 lessons, and the Topic Assessments (pp. 863-864A).

Mathematical Background: Read Cluster Overview- (TE, pp. 815A-815F)	**Topic Essential Questions:** How can you classify triangles and quadrilaterals? What is line symmetry? *Reference TE, p. 815 and Answering the Topic Essential Questions (TE, pp. 861-862) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

16-1	16-2	16-3	16-4	16-5	16-6	Assessment

4 F/D/E days used strategically throughout the topic.

Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Instructional note:
This topic focuses on lines, angles and shapes. Focus for standard 4.G.A, "Draw and identify lines and angles, and classify shapes by properties of their lines and angles" (Nevada Academic Content Standards (NVACS), 2010). Standards 4.G.A.1-3 focus on 1) drawing points, lines, line segments, rays, angles and perpendicular and parallel lines in two-dimensional figures, 2) classifying two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size and 3) recognizing lines of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts (NVACS, 2010).

Van de Walle, Karp, Lovin, Bay-Williams (2014) explain, "Geometry is much more than vocabulary and naming shapes, it is the mathematical study of spatial objects, relationships and movements" (p. 345). There are two concepts in geometry; natural and formal. ***Natural concepts*** are formed during everyday activity and are rarely accompanied by concept definitions. Concepts are usually induced from instances and are thought about in terms of visual resemblances to prototypical examples. ***Formal concepts*** are concepts that have definitions that explicitly specify a sufficient set of properties to identify instances. Before being in school, natural concepts are formed when students learn to identify but not define shapes, natural concepts are formed. The names used for natural and formal concepts may be the same, but the underlying cognitive entities are vastly different. For instance, students who may know a natural concept of a square think of a particular image, but formally think of it as a shape that possesses a specific set of properties. Conceptual discussions around geometric properties and exposure to a variety of examples can support students' understanding of formal concepts in geometry.

Trapezoids include exclusive (E) and inclusive (I) definitions. Below are different definitions for trapezoids:

- T(E): a trapezoid is a quadrilateral with exactly one pair of parallel sides (definition used by enVisionmath2.0, GoMath for district alignment)
- T(I): a trapezoid is a quadrilateral with at least one pair of parallel sides.

The Geometry Progression Documents point out the different meanings result in different classifications at the analytic level. "According to the T(E), a parallelogram is not a trapezoid; according to T(I), a parallelogram is a trapezoid." (2014, p. 3). While the WCSD approved instructional materials supports the T(E) definition (which appears to be the most common definition through high school Geometry); the progression document points out that, "'The preponderance of advantages to the inclusive definition of trapezoid has caused all the articles we could find on the subject, and most college-bound geometry books, to favor the inclusive definition.'" (2014, p. 3).

When working with geometric ideas, be sure to use both prototypical (the typical way that shapes are shown such as the yellow hexagon in a pattern block set); in additional to, non-prototypical examples (any closed figure with 6 connected edges also named a hexagon).

Line of symmetry is an attribute of a shape. Any shape that can be folded in half so that two halves match create a line of symmetry, also known as mirror symmetry. This "line of reflection-portion" of the shape on one side of the line is reflected onto the other side. This demonstrates a connection between line symmetry and transformations. Note: A shape may have multiple lines of symmetry.

In this topic, all images should be considered 2-dimensional representations. That is, we are going to be seeking lines of symmetry based upon the image and not as a reference to the actual object. This will be important to point out to students who may consider the visual images as 3-D instead of the 2-D representations being depicted.

Focus Math Practice 3: **Critique reasoning**
Focus opportunities for students to develop *Mathematical Practice 3* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 16-6. Reference the Teacher's Edition (pp. F23-F23A) and the NVACS (2010, p. 6).

Note: The purpose of the curriculum guides is for additional considerations. Therefore, not all components may have additional notes included in this guide.

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
parallel lines perpendicular lines intersecting lines right triangle obtuse triangle acute triangle equilateral triangle isosceles triangle scalene triangle line symmetric line of symmetry	*parallelogram* *rectangle* *square* *rhombus* *trapezoid*

Additional terminology that students may need support with: geometric term, generalize, plane, always true, sometimes true, never true

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students able to describe various shapes using accurate mathematical vocabulary?"

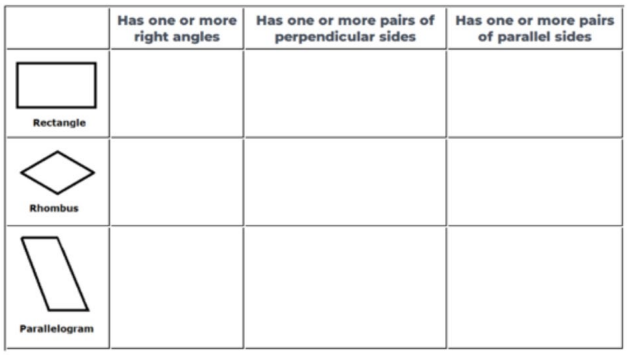
Lesson	Evidence	Look for
16-1	***Math Practice and Problem Solving*** (student work samples) Item 19	Focus CTC on the big idea: • students describe the relationships between different types of lines through use of vocabulary.
16-3	***Quick Check*** (digital platform) Items 1, 2 and 5	Focus CTC on the big idea: • students determine the quadrilateral based on the given attributes Printable version available under "Teacher Resources".

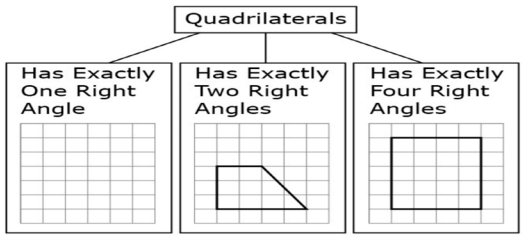
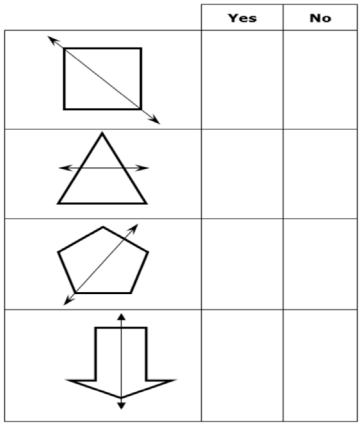
Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 861-864	Use *Scoring Guide* TE pp. 861-864A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Math Practices)	**Mathematical Development of the Big Idea**	**Instructional Clarifications & Considerations**
Lesson 16-1: Lines		
4.G.A.1 **MP.3** MP.4 MP.6	**Access Prior Learning**: In the previous grade, students used the terms parallel and perpendicular to describe attributes of polygons, particularly quadrilaterals. In Topic 15, students learned that a line is a straight path of points that goes on and on in opposite directions. **Developing the Big Idea:** In this lesson, students learn about parallel, perpendicular and intersecting lines.	**Visual Learning:** The *Visual Learning Animation* focus is on mathematical terminology of lines; parallel, perpendicular and intersecting. Consider making an anchor chart with the terminology and pictures of the lines. In the *Visual Learning Animation*, "railroad" example, the perpendicular lines are across two planes. Consider drawing the railroad to visualize it across one plane. **Convince Me:** Consider having students find examples of parallel, intersecting and perpendicular lines in or outside the classroom. Consider having students make a sketch or write a description of where and what was found based on the type of lines. Students may find examples across planes. For example, the corner of the classroom is across two planes. **Guided Practice:** In the *Guided Practice*, items 5 and 6, consider facilitating a discussion that parallel and perpendicular lines must be named in pairs. ***-continues on next page-***

		Assess and Differentiate/Intervention Activity: Consider having all students complete the *Intervention Activity*. After students have completed the *Intervention Activity*, have them make their own map of the streets around their neighborhood of parallel, intersecting and perpendicular lines. Have them include the names of the streets. Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** Select in the box that matches each figure with its description. Each figure may be matched to more than one description. ***CTC:** ***Math Practices and Problem Solving item 19*** (student work samples)
Lesson 16-2: Classify Triangles		
4.G.A.2 **MP.3** MP.6 MP.8	**Access Prior Learning**: In third grade, students classified shapes, including triangles, by looking at their attributes such as the length of the sides or whether or not they have a right angle. In Topic 15, students classified angles as right, acute, obtuse or straight. **Developing the Big Idea:** In this lesson, students classify triangles by their sides and by their angles.	**Note:** Throughout the rest of the lessons, consider bringing in the language of "always true, sometimes true and never true". **Solve & Share:** Consider having tools available for students to use as they complete the *Solve & Share*. **Visual Learning:** In the *Visual Learning Animation*, mathematical terminology is introduced. Consider having students connect back to the sort completed in the *Solve & Share* to see if students can name the different triangles based on what was shown in the animation. **Convince Me:** Consider facilitating a discussion around the *Convince Me!*. Have students use tools to determine if a triangle can have more than one obtuse angle. Consider having students explain their reasoning using mathematical language. Here is an opportunity to use the language, "always true, sometimes true, and never true." **Guided Practice:** Consider facilitating a discussion around item 2, as it connects to the work done in the *Convince Me!*. **Independent Practice/Math Practices and Problem Solving:** Consider reminding students to name a triangle based on both the sides and angles, not just one attribute.
Lesson 16-3: Classify Quadrilaterals		
4.G.A.2 MP.2 **MP.3** MP.6 MP.7 MP.8	**Access Prior Learning**: In third grade, students learned that rhombuses, rectangles, squares, parallelograms and trapezoids are all examples of quadrilaterals. They learned attributes of these different quadrilaterals. **Developing the Big Idea:** In this lesson, students continue to classify quadrilaterals by their attributes.	**Note:** Consider using the mathematical terms; quadrilaterals (sides) and quadrangles (angles) when discussing this lesson as shapes can be described by both their sides and angles. **Solve & Share:** Consider having students complete the *Solve & Share*, or have students sort different quadrilaterals or quadrangles based on the attributes of the quadrilaterals and/or quadrangles. **Look Back:** Consider combining the *Look Back!* with the *Solve & Share* so students can explain the attributes of the quadrilaterals and/or quadrangles and how it relates to the sort. **Visual Learning:** The *Visual Learning Animation* discusses quadrilaterals; consider making an anchor chart with the various quadrilaterals and adding the language quadrangles as well. ***-continues on next page-***



		Convince Me: As students work on the *Convince Me!*, consider having some rectangles and parallelograms available for students to use to help them decide the similarities and differences between the two types of quadrilaterals and/or quadrangles. **Another Example:** Consider comparing the trapezoid from the *Visual Learning Animation* to the trapezoid in the *Another Example!*. Students need exposure to different trapezoids and understand there are two definitions of a trapezoid. See Instructional Note for details regarding the definitions. Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** This chart shows one way to classify quadrilaterals. Use the Connect Line tool to draw a quadrilateral that belongs in the box labeled "Has Exactly One Right Angle."  ***CTC:** ***Quick Check items 1, 2 and 5*** (digital platform)
Lesson 16-4: Line Symmetry		
4.G.A.3 MP.2 MP.3 MP.5 MP.7	**Access Prior Learning**: In the previous grades, students composed and decomposed plane figures to build an understanding of properties of original composite shapes. **Beginning of the Big Idea:** In this lesson, students analyze figures for line symmetry.	**Guided Practice:** Read the Error Intervention before teaching the lesson (TE, pp. 841-842). Consider facilitating a discussion around item 3 in the *Guided Practice***;** how many lines of symmetry does a circle have. Consider having students figure out how many lines of symmetry a circle has by folding a circle. **Homework & Practice:** Consider facilitating a discussion whole group around item 15 of the *Homework & Practice*, as students decide how many lines of symmetry the picture of a wagon wheel has. This may develop into an in-depth conversation about circles and the infinite number of line of symmetry. Consider utilizing this question format during practice: Example 1 Full Statement **Example Stem:** Decide if the line appears to be a line of symmetry for the shape. Select Yes or No for each shape. 

Lesson 16-5: Draw Shapes with Line Symmetry		
4.G.A.3 MP.1 MP.2 **MP.3** MP.4	**Access Prior Learning**: In the previous lesson, students learned how to find one or more lines of symmetry in a two-dimensional figure and learned what it means for a figure to be line symmetric. **Developing the Big Idea:** In this lesson, students learn to draw figures with a given number of lines of symmetry.	**Visual Learning:** Read the Prevent Misconceptions prior to the lesson (TE, p. 846). While students may generate various types of ***transformations (reflection, translation, rotation and dilation),*** explicit instruction for this idea is in later grades. Consider acknowledging the vocabulary used for other types of transformations while reinforcing the idea that ***reflection*** is specifically used for symmetry. **Convince Me:** Consider facilitating a discussion around the *Convince Me!* as students find line of symmetry across a diagonal line. Consider having students actually fold the shape across a diagonal line. Provide additional examples for students to understand conceptually that lines of symmetry do not always reflect across a vertical or horizontal line.
Lesson 16-6: Math Practices and Problem Solving: Critique Reasoning		
4.G.A.2 **MP.3** MP.2 MP.6 MP.7	**Access Prior Learning**: In previous topics and lessons, students have used MP.3 to critique the reasoning of others. **Developing the Big Idea:** In this lesson, students will focus on thinking habits good problem solvers use when they critique the reasoning of others related to analyzing two-dimensional figures.	**Look Back:** Consider facilitating a discussion around the *Look Back!*, as students discover that things can be true without always being true. Challenge the students to draw an example and a non-example. **Note:** In the *Vocabulary Review*, students are asked to identify if claims as being "always, sometimes or never true". Consider using this language throughout this lesson.

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Geometry.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

Council of Chief State School Officers. (2010). The Nevada Academic Content Standards. Retrieved from http://www.doe.nv.gov/uploadedFiles/nde.doe.nv.gov/content/Standards_Instructional_Support/Nevada_Academic_Standards/Math_Documents/mathstandards.pdf.

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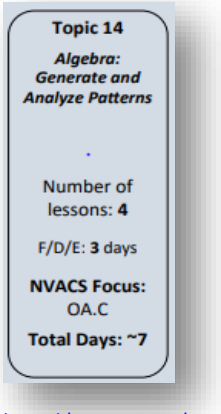
Grade 4 Topic 14: Algebra: Generate and Analyze Patterns



Pacing guides are posted on the **C&I Website** & Teams **Teacher Communities**

Big Conceptual Idea: Operations and Algebraic Thinking (pp. 46-48)
Prior to instruction, view the Topic 14 Professional Development Video located in Pearson Realize online. Read the Teacher's Edition (TE): Cluster Overview/Math Background (pp. 729A-729F), the Topic Planner (pp. 729I-729J), all 4 lessons, and the Topic Assessments (pp.763-763A).

Mathematical Background: Read Cluster Overview- (TE, pp. 729A-729F)	**Topic Essential Questions:** How can you use a rule to continue a pattern? How can you use a table to extend a pattern? How can you use a repeating pattern to predict a shape? *Reference TE (p. 729) and Answering the Topic Essential Questions (TE, pp. 761-762) for key elements of answers to the Essential Questions.*

The lesson map for this topic is as follows:

14-1	14-2	14-3	14-4	Assessment

3 F/D/E days used strategically throughout the topic.

Instructional note:
This topic focuses on generating and analyzing patterns. Focus for standard 4.OA.C.5, "generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself" (Nevada Academic Content Standards (NVACS), 2010). Van de Walle, Karen Karp and Jennifer Bay-Williams (2010) state, "patterns are found in all areas of mathematics. Learning to look for patterns and how to describe, translate and extend them is part of thinking algebraically" (p. 267).

There are different types of patterns: repeated, growing and functional thinking. The focus for this topic will be on repeated and growth patterns. **Repeated patterns** identify the core of the pattern (**Core** is the string of elements that repeats). Students use knowledge of the core to extend the pattern and lays the foundation for the idea that two very different situations can have the same mathematical features. Prediction is an important part of algebraic thinking. (Van de Walle, et al., 2010). **Growing patterns** is a known sequence. In growing patterns, students look for generalizations or algebraic relationships. Students try to determine how each step in the pattern differs from the preceding step. Growing patterns also have a numeric component-the number of objects in each step. This patterning demonstrates the concept of function and can be used as an entry point for this mathematical idea. (Van de Walle, et al., 2010).

"Students are surrounded by patterns in the world around them. Keep a look out for patterns that can be analyzed and used to make predictions. Encourage students to do the same" (Van de Walle, et al., 2014, p. 304).

Focus Math Practice 7: **Look for and use structure**
Consider focusing on opportunities for students to develop *Mathematical Practice 7* behaviors, as this is the focus of the Math Practices and Problem Solving, lesson 14-4. Reference the Teacher's Edition (pp. F27-F27A) and the NVACS (2010, p. 8).

Essential Academic Vocabulary Use these words consistently during instruction.	
New Academic Vocabulary: (First time explicitly taught)	**Review Academic Vocabulary**: (Vocabulary explicitly taught in prior grades or topics)
rule repeating pattern	*growing patterns*

Additional terminology that students may need support with: multiples

***Collaborative Team Conversations (CTC)**
Consider using ***one*** of the following as part of the formative assessment process at the lesson level to **collect student work** to analyze for **evidence of mathematical understanding**:

Guiding question: "Are students able to determine the pattern based on a rule?"

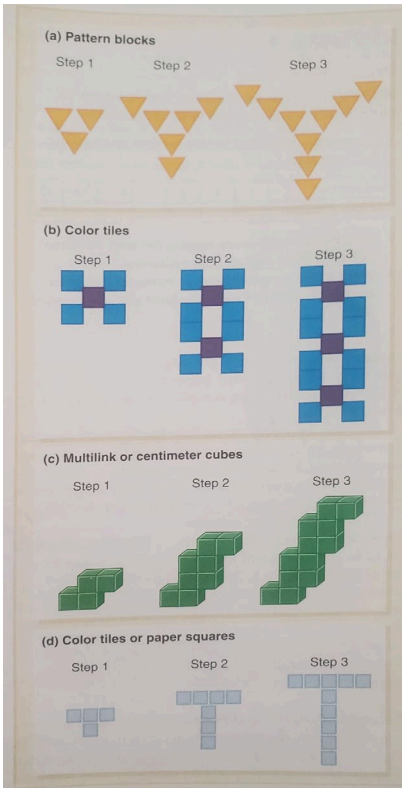
Lesson	Evidence	Look for
14-2	***Quick Check*** (digital platform)	Focus CTC on the big idea: • students determine or use the rule to find patterns Printable version available under "Teacher Resources".
14-4	***Solve & Share*** (student work samples)	Focus CTC on the big idea: • students determine the nth stack by applying the rule and looking for patterns.

Learning Cycle Assessments (summative)	***Topic Assessments*** SE pp. 761-764	Use *Scoring Guide* TE pp. 761-764A

Standards listed in **bold** indicate a focus of the lesson.

NVACS (Content and Math Practices)	Mathematical Development of the Big Idea	Instructional Clarifications & Considerations
Lesson 14-1: Number Sequences		
4.OA.C.5 MP.1 MP.2 MP.4 MP.5 **MP.7** MP.8	**Access Prior Learning**: In Topic 7, students found patterns in multiples and factors, and generalized a rule for determining when all the factors of a number had been listed. **Developing the Big Idea:** In this lesson, students use a given rule to extend a number sequence and then find features of the pattern in the sequence that are not given in the rule.	**Solve & Share:** Considering giving students the opportunity to use tools or representations to support them in the *Solve & Share*. **Look Back:** Consider having students complete the *Look Back!*, as they create a rule and find patterns to fit those rules. **Visual Learning:** In the *Visual Learning*, students learn to use a number line to help find patterns. The problem is an increasing pattern. Read the *Prevent Misconception* before teaching the lesson (TE, p. 734). **Another Example:** In the *Another Example!*, students understand and clarify any misconceptions that patterns only increase, as students find decreasing patterns in this problem. Consider modifying and utilizing the following example in a whole class discussion: Example 1 Full Statement **Example Stem:** A pattern is generated using this rule: Start with the number 7 as the first term and add 5. **Part A:** Drag numbers into the boxes to show the next six terms of this pattern. 7 | | | | | | | First term | Second term | Third term | Fourth term | Fifth term | Sixth term | Seventh term **Part B:** Based on what you observe about the first seven terms, which numbers below are also in the pattern? Select **all** of the numbers that are in the pattern. 377 955 1022 9992
Lesson 14-2: Patterns: Number Rules		
4.OA.C.5 MP.2 **MP.7**	**Access Prior Learning**: In the previous lesson, students found patterns from a given rule. Students then found features of the pattern that were not given in the rule. **Developing the Big Idea:** In this lesson, students generate a table of ordered pairs from a given rule and look for features of the pattern in the table.	**Solve & Share:** Consider removing the rule or the table, and "Use the rule to complete the table" from the *Solve & Share* to elicit more student strategies and increase the cognitive demand. **Look Back:** Consider having students work on the *Look Back!* as partners or small groups. Consider using a Gallery Walk (ELL Toolkit p. 22), to enable students to critique the reasoning of others and use mathematical language when explaining their reasoning. **Independent Practice/Math Practices and Problem Solving:** Consider taking some of the tables in the *Independent Practice* and turning them into problems where students determine the rule and the relationships between the features in the problem. ***-continues on next page-***

		For example, change item 5 from finding the rule and relationship between the numbers of books and weight of the books in ounces to, "The FedEx driver has to carry a box full of books. One book weighs 16 ounces. Two books weigh 32 ounces. How much would the FedEx driver's box weigh if the driver has to carry 10 books? 25 books?" Students determine how to organize the information and find the pattern. Consider utilizing this question format during practice: **Example 1** Full Statement **Example Stem:** A pattern is generated using this rule: Start with the number 7 as the first term and add 5. Enter numbers into the boxes to complete the table. **Term** / **Number** First: 7 Second: Third: Fourth: Fifth: ***CTC: *Quick Check*** (digital platform)
Lesson 14-3: Patterns- Repeating Shapes		
4.OA.C.5 MP.2 MP.3 MP.6 **MP.7**	**Access Prior Learning**: In previous grades, students identified and reasoned with shapes and their attributes. In Lesson 14-1, students learned to extend number patterns, following a given rule. **Developing the Big Idea:** In this lesson, students extend shape patterns following a given repeating pattern by interpreting remainders.	**Solve & Share:** Consider giving students an opportunity to use tools or representations to support them when completing the *Solve & Share*. Child-watch for students who do every single repeated pattern to the 37th shape or have a strategy to find the 37th shape. As an extension for early finishers, have students predict what the 60th shape would be using the rule. **Visual Learning:** Prior to students watching the animation, have students work on the problem. Encourage the use of pattern blocks or reasoning strategies. Pause the *Visual Learning Animation* prior to part C "Use the Repeating Pattern to Solve" or discuss strategies if students did not come up with dividing by 3. In dividing by 3, students solve a measurement division problem with interpreting the remainder of what shapes are included and what are not. The quotient can be interpreted as 16 groups and 1 more shape. Ask students, "Can we use division to solve this problem?" Connect the use of pattern blocks with the statement on Part C (TE, p. 746f). **Convince Me:** Consider having students work on the *Convince Me!* with partners to solve the problem. Have students use pattern blocks and division. **Another Example:** In the *Another Example!*, students figure out the 100th number in the pattern by using division. Again, this is a measurement division problem with no remainder. Consider asking students to find the 115th term, where they will have to interpret a remainder. **Independent Practice/Math Practice and Problem Solving:** Consider having students work on items 8 and 9, as students can use tools or strategies to solve the problems. Facilitate a discussion around these items and students' responses. **Homework & Practice:** Consider using item 9 in the *Homework & Practice* as an extension to solving for patterns. Considering having students work with partners or use as a Gallery Walk.
Lesson 14-4: Math Practices and Problem Solving- Look For and Use Structure		
4.OA.C.5 **MP.7** MP.1 MP.2	**Access Prior Learning**: In previous topics, students had the opportunity to look for and use structure. **Developing the Big Idea:** In this lesson, students continue look for and use structure to find growth patterns.	**Solve & Share:** Consider modifying the *Solve & Share* to "Evan's baby brother is stacking blocks. How many blocks are in the 6th stack? How many blocks are in the 19th stack?" By modifying the problem, this may elicit more student strategies or models. Consider having tools and representations available for students to use when solving the growing pattern, such as connecting cubes or graph paper. **Look Back:** Consider facilitating a discussion around the *Look Back!* as students use the rule found in the *Solve & Share* to figure out the 10th stack of blocks. ***-continues on next page-***

		Guided Practice: Consider taking the triangle pattern from the *Guided Practice,* put it on the board and have students solve the growing pattern. Do not include the table, as you use this opportunity to child-watch students to see how they organize the information. Consider facilitating a discussion around what the students determined about the growing pattern. **Independent Practice/Math Practices and Problem Solving:** Consider choosing items 4-6 or 7-10 to have students solve for growing patterns. Consider also having students come up with their own growing pattern to give their peers to figure out the rule and pattern. Below are more examples of growing patterns using manipulatives from the book *Elementary & Middle School Mathematics (pp. 269).* Consider choosing one for a whole group discussion or use them all in small group rotations. Have students determine the nth term and the rule for the growing patterns.  **Assess and Differentiate/Intervention Activity:** Consider using the *Intervention Activity* with all students to reinforce area. By doing this problem, this will support students with *Homework Practice* items 4-7. ***CTC: *Solve & Share*** (student work samples)

References

Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft). K, Counting and Cardinality; Grades K-5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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